Parallel FFT and Parallel Cyclic Convolution Algorithms with Regular Structures and no Processor Intercommunication

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Approach

• Perform a one dimensional Cyclic Convolution using parallel subconvolutions based on the use of Block Pseudo-Circulant Matrices.
• If a DFT is needed, perform the DFT through a Cyclic Convolution.
• Block Pseudo Circulant Matrices arise from the multiplication of Circulant Matrices by Stride Permutations.
• Pre-processing and post-processing stages are needed to compensate for the multiplication by Stride Permutations.
• The length of the sequences, N, should be composite such as: \( N = RS \), \( N = R^M \) or others. There is no need for the factors to be mutually prime.
• Further restrictions are not imposed.
Basic Relation that Lead to Block Pseudocirculant Matrices in the Context of Cyclic Convolution

\[ y = H_N x \]

Example: \( R=4 \)

\[ Y_p = P_{N,4} y = P_{N,4} H_N (P_{N,4}^{-1} P_{N,4}) x \]

\[ Y_p = P_{N,4} y = (P_{N,4} H_N P_{N,4}^{-1}) P_{N,4} x = (H_P) P_{N,4} x \]

Where \( H_N \) is a Circulant Matrix and \( H_p \) is a Block Pseudo Circulant matrix

\[ Y_p = P_{N,4} y_n = Y_p = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} H_0 & S_{N/4} H_3 & S_{N/4} H_2 & S_{N/4} H_1 \\ H_1 & H_0 & S_{N/4} H_3 & S_{N/4} H_2 \\ H_2 & H_1 & H_0 & S_{N/4} H_3 \\ H_3 & H_2 & H_1 & H_0 \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} = (H_P) P_{N,4} x \]

Where \( S_{N/4} \) is a Cyclic Shift Operator
Because of the Block Pseudocirculant Matrices the Cyclic Convolution can now be Computed using Parallel Subconvolutions of Length $N/R$

Direct Realization for $R=4$

Realization for $R=4$ after Factorization of $H_p$
Parallel DFT using parallel Cyclic Subconvolutions

The Bluestein algorithm can be used to write a DFT as a Cyclic Convolution

\[
X[k] = W_N^{k^2} \frac{1}{2} \sum_{n=0}^{N-1} (W_N^{n^2} x[n]) W_N^{-(k-n)^2} = W_N^{k^2} \frac{1}{2} \sum_{n=0}^{N-1} (x_{new}[n]) h[n - k]
\]