# COMPLEMENTARY BEAMFORMING

*Vahid Tarokh* — *Yang-Seok Choi and Siavash M. Alamouti*

Harvard University
33 Oxford Street Room MD-347
Cambridge, MA 02139

Vivato Research and Development
12610 E. Mirabeau Parkway Suite 900
Spokane, WA 99216

## ABSTRACT

***We propose** Complementary Beamforming **(CBF) for wireless communications. A major application of complementary beamforming is in smart antennas enhancements to the IEEE 802.11 systems. In these systems, beamforming may be employed in order to increase the range/capacity. When a conventional beamformer is employed to increase the transmitted power to some specific directions, the radiated power to other directions is reduced. This means that some other users of the system may experience lower signal to noise ratios. During a busy period of the channel, these users may wrongly determine that the channel is idle and start transmitting packets. This may cause unnecessary transmissions, subsequent back-offs, increased network latency and interference. Furthermore, the aforementioned undesired packet transmission has an energy penalty which adversely effects the battery life of the remote devices. This "*Hidden Beam Problem*" is further exuberated if the system is more heavily loaded which will be most likely the case both in hot spots and also whenever the system range is increased. In this work, we apply complementary beamforming techniques and construct techniques that are designed to significantly reduce the probability of the aforementioned collisions, back-offs and retransmissions. We analyze the proposed scheme for both the intended and silent users and prove that, when compared to conventional methods, for a meager incurred power loss for the intended users, the effects of the hidden beam problem can be significantly reduced. Moreover, we will show that the complementary beamforming described in this paper are approximately twice as complex as conventional beamforming techniques. Finally, we will illustrate the proposed technique by some examples.***

## 1. INTRODUCTION

Current IEEE 802.11 wireless LANs support a maximum throughput of 54 Mbps and have a limited range. In fact, channel models developed from extensive measurements performed jointly by AT&T Labs-Research and Harvard University have been used to prove that such a throughput is only achievable in a radius of 8 meters in NLOS (none line-of sight) environments [1, 2]. At present, a "Wi-Fi revolution" is taking place and it is expected that the widespread deployment of Wi-Fi will change the entire wireless landscape in few years. Abundance of data hungry users in *hot spots* will motivate wireless LAN providers to seek WLAN devices with increased throughputs and ranges. This has created significant interest and activities in the wireless industry focusing on techniques to increase the range and capacity of Wi-Fi networks.

An enhancement that seems to provide an appealing solution is the use of antenna arrays at access points (AP) in conjunction with beamforming. Such a solution is appealing as it is transparent to receivers and does not force any changes to current standards. In spite of these benefits, design of beamforming enhancement to WLANs is not as trivial as the channel sensing mechanisms employed in standard Wi-Fi equipments produces a host of new problems when combined with beamforming.

## 2. THE HIDDEN BEAM PROBLEM

To illustrate this problem, let us consider a scenario when a system employs $m$ transmit antennas and the transmitter simultaneously transmits to $k$ users. Without loss of generality, we assume that $k \leq m$.

A conventional beamformer seeks to increase the power pointed to the $k$ desired users. Consider a scenario where there are $m = 2$ transmit antennas and $k = 1$ intended users. Let the channel matrix to the desired user be given by $(\alpha, \beta)$. A conventional beamformer then induces weigths $w_1 = \frac{\bar{\alpha}}{\sqrt{|\alpha|^2+|\beta|^2}}$ and $w_2 = \frac{\bar{\beta}}{\sqrt{|\alpha|^2+|\beta|^2}}$ at the transmitter, where $\bar{\alpha}$ and $\bar{\beta}$ are the conjugates of $\alpha$ and $\beta$ respectively.

If $c_1$ is the intended transmit signal at time 1 for user 1, then $w_1 c_1$ and $w_2 c_1$ are transmitted signals from antennas 1 and 2 respectively. The intended user receives the signal

$$r_1 = w_1 \alpha_1 c_1 + w_2 \alpha_2 c_1 + n_1 = \sqrt{|\alpha|^2 + |\beta|^2} c_1 + n_1, \quad (1)$$

where $n_1$ is the noise. It is immediately observed that the signal to noise power ratio of the desired user improves by a factor of $10\log_{10}(|\alpha|^2 + |\beta|^2)$ dB.

The above power improvement does not come for free. Let an unintended user have channel matrix $(-\bar{\beta}, \bar{\alpha})$. Then the signal at this unintended user is given by

$$y_1 = -\bar{\beta} w_1 c_1 + \bar{\alpha} w_2 c_1 + \eta_1 = \eta_1, \quad (2)$$

where $\eta_1$ is the noise vector and the unintended user receives no signal.

We observe that there is no part of the transmitted signal present at this unintended user's receiver. This by itself may not seem to pose a serious problem, since after all the transmission was not intended for this user. But it turns out that it can cause a problem in beamforming enhancements to CSMA based systems such as those based on the IEEE 802.11 WLAN standard. In these systems, all users and the access point share the same channel for both up-link and down-link transmissions. Each user senses the channel and only transmits packets if it determines that the channel is not busy. The unintended receiver who does not receive a strong signal component may wrongly determine that the channel is idle and start transmitting packets. This may cause unnecessary transmissions, subsequent back-offs, increased network latency and interference. Furthermore, the aforementioned undesired packet transmission has an energy penalty which adversely effects the battery life of the remote devices. This "*Hidden Beam Problem*" is further exuberated if the system is more heavily loaded which will be most likely the case both in hot spots and also whenever the system range is increased. We refer to this problem as *the hidden beam problem*.

## 3. COMPLEMENTARY BEAMFORMING

The main intuition behind complementary beamforming is that much less power is needed for an unintended user to correctly detect a busy period than that required for correct detection of the transmitted packet. The above observation is built in the detection criteria for channel activity in IEEE 802.11 WLAN standards. Each device listens to the channel during some time window and compares the energy collected in this window to a value called the CCA (Clear Channel Assessment) threshold. Activity is detected only if the collected energy is greater than the CCA threshold.

From the above, we observe that any IEEE 802.11 device requires much less receive power to correctly determine channel activity than to decode the transmitted signals. This motivates our solution to the hidden beam problem. We will seek to construct a beam pattern which directs most of the transmitted power to the intended recipients while directing a small fraction of the total power to unintended users. Once such a beam pattern is designed, the unintended users will all sense the transmission to the desired users with high probability and will keep silent during a busy down-link period. This in turn reduces the packet collision probability.

Next we construct such a beam pattern. To this end, we first introduce some notations.

**Notation:**

- $\delta_j$ denotes a $k$-dimensional column vector with $j$-th component equal to 1 and other components equal to zero.
- For any vector $X$, we let $X^T$ and $X^H$ respectively denote the transpose and Hermitian of $X$.
- For any matrix $D$, we let $W_D$ denote the vector space spanned by the columns of $D$.
- Let the channel from transmit antenna $l$ to the intended user $j$ be given by $\alpha_{l,j}$.
- Let $A_j$ denote the column vector $(\alpha_{1,j}, \cdots, \alpha_{m,j})^T$. We refer to the vector $A_j$ as the *spatial signature of user $j$*.
- Let $A$ denote the matrix whose $j$-th column is $A_j$.
- Let $R^t = (r_1^t, \cdots, r_k^t)$ and $X^t = (x_1^t, \cdots, x_m^t)$ respectively denote the vector of received signals at intended users $j = 1, 2, \cdots, k$ and the vector of signals transmitted from antennas $1, 2, \cdots, m$ at time $t$.
- Let $C^t = (c_1^t, \cdots, c_k^t)$, where $c_j^t$ is the signal intended to the $j = 1, 2, \cdots, k$ desired user at time $t$.
- For any square matrix $A$, let $Tr(A)$ denotes the trace (sum of diagonal elements of $A$).
- Let $N^t = (n_1^t, \cdots, n_m^t)$ be the noise vector components at time $t$ at the intended users.

Then it is well-known that

$$R^t = X^t A + N^t, \quad (3)$$

In most cases, these noise components are assumed to be i.i.d. Gaussian with variance $\sigma^2$ per complex dimension. We make *absolutely no assumptions* on the statistics of the matrix $A$.

It will be assumed that $c_j^t$, $j = 1, \cdots, k$, $t = 1, \cdots, L$ are elements of a signal constellation with average signal $E[c_j^t] = 0$. We will also assume that the elements of the signal constellation are normalized so that their average power is $E[|c_j^t|^2] = 1$.

In general $X^t = C^t \mathcal{B}$ where $\mathcal{B}$ is referred to as the *beamforming matrix*. The choice of $\mathcal{B}$ depends on the beamforming strategy and many approaches for the selection of $\mathcal{B}$ are suggested in the literature. Assuming that the matrix $A$

is known at the transmitter and the existence of $(A^H A)^{-1}$, for a zero-forcing beamformer

$$\mathcal{B} = \frac{(A^H A)^{-1} A^H}{\sqrt{Tr((A^H A)^{-1})}}$$

and

$$X^t = \frac{C^t (A^H A)^{-1} A^H}{\sqrt{Tr((A^H A)^{-1})}}. \quad (4)$$

Another commonly used beamforming matrix is given by

$$\mathcal{B} = \frac{(A^H A + \frac{1}{\text{SNR}} I)^{-1} A^H}{\sqrt{Tr((A^H A + \frac{1}{\text{SNR}} I)^{-2} A^H A)}}, \quad (5)$$

where $SNR = \frac{1}{\sigma^2}$. Other choices of beamforming matrices are also possible.

Under the above assumptions the total transmit power is easily computed to be 1. For simplicity, we present our technique for the zero-forcing beamformer here. Nonetheless, we note that the method that we present here generalizes to other cases as well. In the following, we will sometimes describe this generalization. Moreover in our presentation, we also assume that the spatial signature matrix $A$ is constant during the transmission of a packet and varies from one packet to another.

Assuming a zero-forcing beamformer, the received signal at the receiver is given by

$$R^t = \frac{C^t}{\sqrt{Tr((A^H A)^{-1})}} + N^t,$$

and we observe that each intended user $j = 1, \cdots, k$ receives a noisy version of its intended signal scaled by a factor $Tr((A^H A)^{-1})$.

It is readily observed that if an unintended user has spatial signature $B = (b_1, b_2, \cdots, b_m)^T$ orthogonal to all the rows of $A$, then it receives the signal

$$y^t = X^t B + \eta^t = \eta^t,$$

at time $t$, where $\eta^t$ is Gaussian noise. This means that such a user does not receive any signal components. As mentioned above, such an unintended user can confuse a busy down-link period with a silent period and transmit packets during a busy period. This can cause unwanted collisions and reduce the efficiency of the system. Thus we arrive at a simple albeit important conclusion that, *whenever a $k \times m$ beamforming matrix is fixed during transmission of a packet, then any unintended user that has spatial signature in the orthogonal complement of the subspace generated by the rows of the beamforming matrix receives no signal components*. In time domain, this motivates the use of different beamforming matrices at different instances of time during the transmission of down-link packets, so that the effects of the hidden beam problem can be reduced.

### 3.1. The Proposed Scheme:

The subspace $W_A$ is a $k$-dimensional subspace of the complex $m$-dimensional complex space and has an orthogonal complement $W_A^\perp$ of dimension $m-k$. Let $U_0, \cdots, U_{m-k-1}$ form an orthonormal basis for $W_A^\perp$. In other words, $U_0, U_1, \cdots, U_{m-k-1}$ are mutually orthogonal $m$-dimensional column vectors of length one in $W_A^\perp$. Clearly, $U_j^H A_i = 0$ for $0 \le j \le m-k-1$ and $1 \le i \le k$.

First, the transmitter constructs matrices $Z_1, Z_2, \cdots, Z_L$, where $L$ is the the length of down-link transmission period, such that these matrices satisfy the following properties.

- **A:** For all $1 \le i \le L$, the matrix $Z_i$ is a $k \times m$ matrix whose rows are in the set

  $$\{0, \pm U_0^H, \pm U_1^H, \cdots, \pm U_{m-k-1}^H\},$$

- **B:** For $L$ even, $Z_2 = -Z_1$, $Z_4 = -Z_3, \cdots, Z_L = -Z_{L-1}$,
- **C:** For $L$ odd, $Z_2 = -Z_1$, $Z_4 = -Z_3, \cdots, Z_{L-1} = -Z_{L-2}$, $Z_L = 0$, and
- **D:** Each element

  $$+U_0^H, -U_0^H, +U_1^H, -U_1^H, \cdots, +U_{m-k-1}^H, -U_{m-k-1}^H$$

  appears $p$ times in the the list of $Lk$ rows of $Z_1, \cdots, Z_L$ for some positive integer $p$. If this cannot be exactly satisfied, we try to have the number of these appearances as large and as close as possible to each other. Clearly, for $p = \lfloor k(L-1)/2(m-k) \rfloor$, it is possible to have $p$ occurrences of each of these vectors and $r$ occurrences of the zero vector, where $r = LK - 2p(m-k)$.

From Property **D**, it is immediately observed that

$$kL - 2m - k \le 2p(m-k) \le 2\lfloor \frac{L}{2} \rfloor k. \quad (6)$$

Because $p \ge 1$, from the above inequality, we observe that for $L < \frac{2(m-k)}{k}$, Property **D** cannot be exactly satisfied. Thus, for extremely short packets, we cannot always provide a perfectly balanced appearance of $+U_0^H, -U_0^H, +U_1^H, -U_1^H, \cdots, +U_{m-k-1}^H, -U_{m-k-1}^H$

In practice, there are a number of easy ways to implement construction of matrices of $Z_1, Z_2, \cdots, Z_L$ that approximately or exactly satisfy Property (**D**) [3]. Once $Z_1, Z_2, \cdots, Z_L$ are constructed, at each time $t$, the transmitter chooses the beamforming matrix

$$S^t = [((A^H A)^{-1} A^H / \sqrt{Tr((A^H A)^{-1})} + \frac{1}{\sqrt{k}} \epsilon Z_t], \quad (7)$$

where $\epsilon \geq 0$ is a fixed positive number. The choice of $\epsilon \geq 0$ governs the trade-off between the power pointed to the intended users and that pointed to unintended users. By increasing the power pointed to intended users, the intended users enjoy better channels, while by pointing more power to unintended users, better channel activity detection during the busy periods can be achieved. This trade-off will be analyzed in the next section and criteria for the choice of $\epsilon \geq 0$ will be determined. For $\epsilon = 0$, we recover the conventional beamforming. Thus complementary beamforming generalizes and includes conventional beamforming as a special case.

We note that in the proposed scheme the beamforming matrix varies from one time to another. This guarantees that a small fraction of power is pointed to every direction of the space and that the unintended receivers can determine channel activity periods with higherprobabilities.

## 4. ANALYSIS OF COMPLEMENTARY BEAMFORMING

We analyze complementary beamforming scheme both for the intended and unintended receivers.

### 4.1. The Power Penalty for The Intended Users:

The addition of the term $\frac{1}{\sqrt{k}}\epsilon Z_i$ to the matrix $(A^H A)^{-1} A^H / \sqrt{Tr((A^H A)^{-1})}$ increases the transmit power. To compute the penalty, we use the orthogonality of $U_0, U_1, \cdots, U_{m-k-1}$ and the columns of $A$ to conclude that $Z_t A = 0$ for all $t = 1, 2, \cdots, L$. Thus, we compute the receive word for intended users to be

$$R^t = C^t S^t A + N^t = \frac{C^t}{\sqrt{Tr((A^H A)^{-1})}} + N^t,$$

which is the same as the conventional beamforming. In contrast, in the case of complementary beamforming, we use the matrix equality

$$\begin{aligned} &Tr\left[(Y+W)(Y+W)^H\right] + Tr\left[(Y-W)(Y-W)^H\right] \\ &= 2Tr(YY^H) + 2Tr(WW^H), \end{aligned}$$

and Properties **B** and **D** to compute the average transmitted power

$$\frac{\sum_{t=1}^{L} Tr(S_t S_t^H)}{L} = 1 + \frac{\sum_{t=1}^{L} Tr(Z_t Z_t^H)}{Lk} |\epsilon|^2.$$

From Property **D**, we have

$$\sum_{t=1}^{L} Tr(Z_t Z_t^H) = 2p(m-k),$$

thus

$$\frac{\sum_{t=1}^{L} Tr(S_t S_t^H)}{L} = 1 + \frac{2p(m-k)}{Lk} |\epsilon|^2.$$

We can now prove the following Theorem.

**Theorem 1** *The intended users in complementary beamforming when compared to the conventional method suffer a loss of at most* $10\log_{10}(1+|\epsilon|^2)$.

**Proof:** This follows from the above and from Inequality (6).

### 4.2. Analysis of The Power delivered to Silent Users:

Let $B = (b_1, b_2, \cdots, b_t)^T$ denote the channel of an arbitrary unintended user. We will next study the power received by this unintended user under complementary beamforming. To this end, we recognize that the columns of matrix $A$ and the vectors $U_0, U_1, \cdots, U_{m-k-1}$ span the complex $m$-dimensional space. Thus we can write

$$B = \sum_{i=1}^{k} e_i A_i + \sum_{j=1}^{m-k-1} d_j U_j, \tag{8}$$

for some constants $e_1, e_2, \cdots, e_k$ and $d_1, d_2, \cdots, d_{m-k}$. Letting

$$\mathbf{e} = (e_1^H, e_2^H, \cdots, e_k^H), \tag{9}$$

by computing $B^H B$, we arrive at

$$\sum_{j=1}^{m} |b_i|^2 = \mathbf{e} A^H A \mathbf{e}^H + \sum_{j=0}^{m-k-1} |d_j|^2. \tag{10}$$

At time $t$, the unintended receiver now receives

$$y^t = X^t B + \eta^t = C^t S^t B + \eta^t.$$

By replacing for $S^t$ and $B$ from Equations (7) and (8) and observing that

$$\begin{aligned} (A^H A)^{-1} A^H A_j &= \delta_j, \\ A^H U_i &= 0 \\ Z_t A_i &= 0, \end{aligned}$$

we arrive at the conclusion that

$$S^t B = \frac{(e_1^H, e_2^H, \cdots, e_k^H)^H}{\sqrt{Tr((A^H A)^{-1})}} + \frac{\epsilon}{\sqrt{k}} \sum_{j=0}^{m-k-1} d_j Z_t U_j. \tag{11}$$

We next compute the average expected receive signal power

$$P_{av} = \frac{\sum_{t=1}^{L} E[|y^t|^2]}{L} = \frac{\sum_{t=1}^{L} Tr(S^t B B^H (S^t)^H)}{L}. \tag{12}$$

However, since $Z_{2l} = -Z_{2l-1}$ for $l = 1, 2, \cdots, \lfloor \frac{L}{2} \rfloor$ is assumed, we can use Equation (11) and simple manipulations to arrive at

$$\sum_{i=0}^{1} Tr(S^{2l-i} BB^H (S^{2l-i})^H = \frac{2\sum_{j=1}^{k} |e_j|^2}{Tr((A^H A)^{-1})}$$
$$+\frac{|\epsilon|^2}{k} \sum_{j=0}^{m-k-1} |d_j|^2 \sum_{i=0}^{1} Tr(Z_{2l-i} U_j U_j^H Z_{2l-i}^H)$$

Using the above and after simple manipulations, we arrive at

$$P_{av} = K(L) \frac{\sum_{j=1}^{k} |e_j|^2}{Tr((A^H A)^{-1})} + \frac{|\epsilon|^2}{kL} \sum_{j=0}^{m-k-1} |d_j|^2 \sum_{t=1}^{L} Tr(Z_t U_j U_j^H Z_t^H),$$

where $K(L) = 2\lfloor L/2 \rfloor / L$. The sum $\sum_{t=1}^{L} Tr(Z_t U_j U_j^H Z_t^H)$ is exactly equal to the number of times that $\pm U_j$ appears in the list of the rows of $Z_1, Z_2, \cdots, Z_L$. By Property **D** this amounts to $2p$. Thus

$$P_{av} = K(L) \frac{\sum_{j=1}^{k} |e_j|^2}{Tr((A^H A)^{-1})} + |\epsilon|^2 \frac{2p}{kL} \sum_{j=0}^{m-k-1} |d_j|^2. \quad (13)$$

We now proceed to lower bound $P_{av}$. To this end, we prove the following theorem.

**Theorem 2** *Let $\lambda_{\min}(A^H A)$ and $\lambda_{\max}(A^H A)$ respectively denote the minimum and maximum eigenvalues of $A^H A$. Then provided that*

$$|\epsilon|^2 \leq \frac{(m-k)}{k} \frac{\lambda_{\min}(A^H A)}{\lambda_{\max}(A^H A)}, \quad (14)$$
$$p \geq \frac{m}{k} - 0.5, \quad (15)$$

*complementary beamforming gurantees a fraction $|\epsilon|^2 \frac{\sum_{j=1}^{m} |b_j|^2}{m}$ of the transmitted power to an unintended receiver whose spatial signature is $B = (b_1, b_2, \cdots, b_m)$.*

**Proof:** Let an unintended user with spatial signature given by $B = (b_1, b_2, \cdots, b_m)$ be given. Suppose that the Inequality (14) holds. From Equations (10) and (13), we observe that

$$P_{av} = |\epsilon|^2 \frac{2p}{kL} \sum_{i=1}^{m} |b_i|^2 - \mathbf{e} G \mathbf{e}^H,$$

where

$$G = \left[ (|\epsilon|^2 \frac{2p}{kL} A^H A - \frac{K(L) I}{Tr((A^H A)^{-1})} \right],$$

and $I$ is the identity matrix. The matrix $G$ is Hermitian, thus we conclude from the above that

$$P_{av} \geq |\epsilon|^2 \frac{2p}{kL} \sum_{i=1}^{m} |b_i|^2 - \lambda_{\max}(G) \sum_{j=1}^{k} |e_k|^2 \quad (16)$$

where $\lambda_{\max}(G)$ is the maximum eigenvalue of $G$. Clearly

$$\lambda_{\max}(G) = |\epsilon|^2 \frac{2p}{kL} \lambda_{\max}(A^H A) - \frac{K(L)}{Tr((A^H A)^{-1})}.$$

Next, we prove that $\lambda_{\max}(G) \leq 0$. Clearly

$$Tr((A^H A)^{-1}) \leq \frac{k}{\lambda_{\min}(A^H A)},$$

thus using Condition (14)

$$\frac{1}{Tr((A^H A)^{-1})} \geq \frac{\lambda_{\min}(A^H A)}{k} \geq \frac{|\epsilon|^2 \lambda_{\max}(A^H A)}{m-k},$$

which gives

$$|\epsilon|^2 \frac{2p}{kL} \lambda_{\max}(A^H A) \leq \frac{2p(m-k)}{kL} \frac{1}{Tr((A^H A)^{-1})} \leq \frac{K(L)}{Tr((A^H A)^{-1})},$$

where we used Inequality (6). We conclude from the above that $\lambda_{\max}(G) \leq 0$. Using Equation (16), this implies that

$$P_{av} \geq |\epsilon|^2 \frac{2p}{kL} \sum_{i=1}^{m} |b_i|^2 \geq |\epsilon|^2 \frac{2pm}{kL} \frac{\sum_{i=1}^{m} |b_i|^2}{m}. \quad (17)$$

Using the Inequality (6) and the condition $p \geq \frac{m}{k} - 0.5$, we can now conclude that $P_{av} \geq |\epsilon|^2 \frac{\sum_{i=1}^{m} |b_i|^2}{m}$.

**Discussion:** Intuitively, the condition $p \geq \frac{m}{k} - 0.5$ means that the transmitted packets must not be too short. However, it may not seem intuitive to the reader that the Condition (14) on $\epsilon$ contains terms of the form $\lambda_{\min}(A^H A)/\lambda_{\max}(A^H A)$. We argue that this condition is intuitively appealing. To this end, consider the case that the ratio $\lambda_{\min}(A^H A)/\lambda_{\max}(A^H A)$ is small. Then the matrix $A^H A$ is close to being singular. This means that even the intended users, do not receive significant signal powers. In fact, practical beamforming schemes, when scheduling transmission to intended users always assure that the ratio $\lambda_{\min}(A^H A)/\lambda_{\max}(A^H A)$ is not close to zero and in most cases even larger than a pre-specified threshold. In practice, a ratio $\lambda_{\min}(A^H A)/\lambda_{\max}(A^H A) \geq \frac{1}{3}$ is generally an acceptable assumption. In the case of system with $k = 4$, $m = 16$. Thus, provided that scheduling algorithm can guarantee that $\lambda_{\min}(A^H A)/\lambda_{\max}(A^H A) \geq \frac{1}{30}$, the above complementary beamforming scheme could be used to provide any fraction $|\epsilon|^2 \leq 0.1$ of the transmitted power to unintended users.

## 5. EXAMPLES

**Example I:** We consider the case when there are $m = 2$ transmit antennas and $k = 1$ intended receivers. Assuming that the channel to the intended user is given by $A = (\alpha, \beta)^T$, we observe that $\lambda_{\min}(A^H A)/\lambda_{\max}(A^H A) = 1$ and as long as $|\epsilon|^2 \leq 1$, by the above theorema fraction $|\epsilon|^2$ of the transmitted power is pointed to unintended users at the expense of a loss of at most $10 \log_{10}(1 + |\epsilon|^2)$ to the intended user. With $\epsilon = 0.1$, we observe that a power of 20 dB below transmit power can be guaranteed to any unintended users so that they can detect channel activity, while the power penalty for the intended user is only 0.044 dB.

The beamforming matrices $S_1$ and $S_2$ in this case are given by

$$S_1 = \frac{1}{\sqrt{|\alpha|^2 + |\beta^2|}}(\bar{\alpha} - \epsilon\beta, \bar{\beta} + \epsilon\alpha),$$
$$S_2 = \frac{1}{\sqrt{|\alpha|^2 + |\beta^2|}}(\bar{\alpha} + \epsilon\beta, \bar{\beta} - \epsilon\alpha),$$

with $S_{2l-1} = S_1$ and $S_{2l} = S_2$ for $l = 1, 2, \cdots, \lfloor \frac{L}{2} \rfloor$ when the transmission period is of length $L$ with $S_L = \frac{1}{\sqrt{|\alpha|^2+|\beta^2|}}(\bar{\alpha}, \bar{\beta})$ when $L$ is odd.

In Figures 1 and 2, we have compared CBF with conventional beamforming using the above scheme. The distance $d$ between the transmit antennas is set to be half of the wavelength $\lambda$ and $\epsilon = 0.3$. Beam patterns in time are illustrated and the average power value (CBF) is compared to the conventional case. It is immediately seen from the Figures, that with a meager power penalty to the intended user, the hidden beam problem is completely eliminated.

In general, it can be shown that complementary beamforming is approximately twice as much computationally intensive as conventional beamforming [3].

## 6. CONCLUSION

In this paper, we proposed *Complementary beamforming* (CBF) for wireless communications. An application of CBF, outlined in this paper, is in smart antennas enhancements to the IEEE 802.11 systems which suffer from the hidden beam problem. We analyzed the proposed scheme for both the intended and silent users and proved that, when compared to conventional beamforming methods, for a negligible incurred power loss for the intended users, the effects of the hidden beam problem caused by the unintended users in the system can be significantly reduced. The proposed technique was illustrated by some examples We indicated that the complexity of complementary beamforming is approximately twice as much as that of the conventional beamforming.

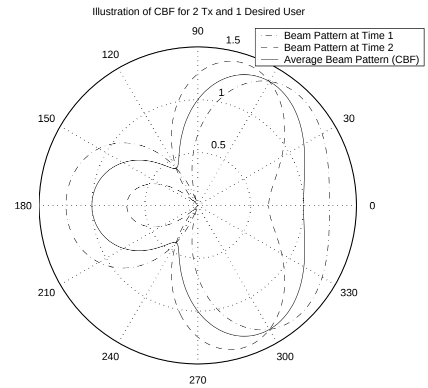


**Fig. 1**. Illustration of Time Domain CBF pointing to $\theta = \frac{\pi}{3}$ and $d = \lambda/2$

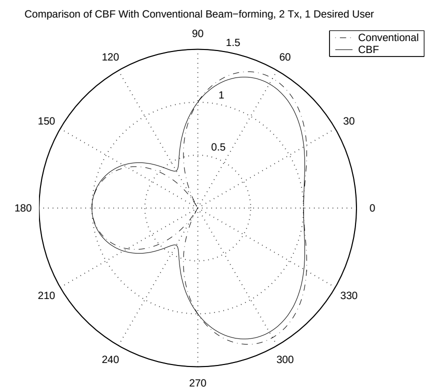


**Fig. 2**. Comparison of CBF and Conventional Beamforming for $\epsilon = 0.3$.

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