Motivation

- Radar targets provide a rich scattering environment.
- Conventional radars experience target fluctuations of 5-25 dB.
- Slow RCS fluctuations (Swerling I model) cause long fades in target RCS, degrading radar performance.
- In statistical MIMO the angular spread of the target backscatter is exploited in a variety of ways to extend the radar’s performance envelope.

Backscatter as a function of azimuth angle, 10-cm wavelength [Skolnik 2003].
The S-MIMO Concept

- Statistical-MIMO radar offers the potential for significant gains:
  - Detection/estimation performance
  - Resolution performance
- Here, we focus only on detection performance
- Our results question the common belief that one should maximize the coherent processing gain.
- With S-MIMO a very sparse array of sensors transmits a set of orthogonal waveforms.
- By using this approach, we create many "independent" radars, that average out target scintillations.
Signal Model

- Point source assumption dominates current models used in radar theory.
- This model is not adequate for an array of sensors with large spacing between the array elements.
- Distributed target model
• Denote by $\alpha_{jk}$ the gain between the $k$th transmitter and $j$th receiver. It can be shown that $\alpha_{jk} \sim CN(0,1)$.

• Take $\alpha_{jk}$ and $\alpha_{il}$. We can show that if either $d_1 > d_1 \lambda_c / d_2$ or $d'_1 > d'_1 \lambda_c / d'_2$, then $E\left\{\alpha_{jk} \alpha_{il}^H\right\} \neq 0$, and otherwise $E\left\{\alpha_{jk} \alpha_{il}^H\right\} = 1$
Phased Array Radar

- Phased array radars consist of closely spaced sensors. The gain between each transmitter receiver pair is the same.

- Transmitted waveform is $s(t)$

- This gives rise to the following received signal model
  \[
  r(t) = \sqrt{\frac{E}{M}} \alpha a(x_0, y_0) b(x_0, y_0)^H s(t - \tau) + n(t)
  \]

- If beamformer is applied at both the transmitter and the receiver, then the received signal at the output of the beamformer equals
  \[
  y(t) = \sqrt{\frac{E}{M}} \alpha \|a(x_0, y_0)\|^2 \|b(x_0, y_0)\|^2 s(t - \tau) + n'(t)
  \]
In S-MIMO radar, the inter element spacing is large. The gain between every transmitter receiver pair is different.

The received signal is given by

\[ r(t) = \sqrt{\frac{E}{M}} Hs(t - \tau) + n(t) \quad \text{vec}(H) \sim \text{CN}(0, I) \]

Each transmitting element transmits one of \( M \) orthogonal waveforms.

By matched filtering the received signal at each sensor with each of the transmitted waveforms we can reconstruct

\[ r_{ji}(t) = \sqrt{\frac{E}{M}} \alpha_{ji} s_i(t - \tau) + n_{ji}(t) \]

Therefore, instead of coherent gain of \( MN \), we created \( MN \) independent radars.
The Radar Detection Problem

- The radar detection problem:
  - $H_0$: Target does not exist at delay $\tau$
  - $H_1$: Target exists at delay $\tau$

- Assume that all the parameters are known. The optimal detector is the LRT detector, and it is given by,

$$ T = \log \frac{f(r(t)|H_1) > H_0}{f(r(t)|H_0) < H_1} $$

S-MIMO Radar

- Denote by $x$ the vector that contains the output of a bank of matched filters sampled at $\tau$. The optimal detector is

$$ T = \|x\|^2 > H_1 \delta, \quad \text{where} \quad \delta = \frac{\sigma_n^2}{2} F_{\chi^2_{2MN}}^{-1}(1-P_{FA})$$
• It is possible to compute the probability of detection as a function of the probability of false alarm, and it equals

\[ P_D = 1 - F_{\chi^2_{2MN}} \left( \frac{\sigma_n^2}{E} F_{\chi^2_{2MN}}^{-1} \left( 1 - P_{FA} \right) \right) + \sigma_n^2 \]

**Phased Array Radar**

• Let \( x = \int r^H(t) a(x_0, y_0) s(t - \tau) \, dt \). The optimal detector:

\[ T = \begin{cases} |x|^2 & \text{if } H_0 \\ < & \text{if } H_1 \end{cases} \]

\[ \delta = \frac{N\sigma_n^2}{2} F_{\chi^2_n}^{-1} \left( 1 - P_{FA} \right) \]

\[ P_D = 1 - F_{\chi^2_n}^{-1} \left( \frac{\sigma_n^2}{\sigma_n^2 + EN} F_{\chi^2_n}^{-1} \left( 1 - P_{FA} \right) \right) \]
The Invariance Detector

- Assume access to a vector $y$ that contains $L$ samples of the noise process.

- Note that $\frac{\|y\|^2}{L}$ is the ML estimate of the noise level.

- The optimal detector whose performance depends only on SNR (not on the noise level)

$$T = \frac{\|x\|^2}{\|y\|^2} >_{H_1} \delta$$

$$\frac{\|y\|^2}{\|y\|^2} <_{H_0} \delta$$

- This test statistic is very intuitive. It normalizes the UMP test by the best estimate of the noise level.
Example: Miss Probability

- Assume a system with four receiving and one or two transmitting antennas, $M=2$, $N=4$, and the probability of false alarm is $1e-6$. 

![Graph showing the relationship between SNR and $P_{MD}$ for different antenna configurations.](image)
Example: ROC

- The following figure depicts the ROC. SNR=10dB.
Concluding Remarks

• S-MIMO is a new concept for radar systems.

• This concept utilizes spatial diversity in order to overcome target scintillations.

• At 90% probability of detection, the proposed system outperform phased array radars by 5 dB, which is equivalent to almost twice the range.

• The S-MIMO radar can be shown to have superior performance in range estimation and resolution as well.