Multiuser MIMO Systems

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Outline

• MIMO Multiuser Detection (MUD)
  – Space-time MUD: The SIMO Case
  – MIMO MUD: Examples

• Network Utility in Multiuser MIMO Systems

• Spectral Efficiency in Multiuser MIMO Systems (Briefly)
SPACE-TIME MUD (The SIMO Case)

Multiuser MIMO Systems
Multipath SIMO MAC

User 1: 010...
User 2: 110...
User K: 011...

Multiuser MIMO Systems
Sufficient Statistic
(Space-Time Matched Filter Bank)

\[ K \text{ Users; } P \text{ Receive Antennas; } L \text{ Paths/User/Antenna} \]

\[ r_1(t) \quad r_2(t) \quad \vdots \quad r_P(t) \]

Temporal Matched Filters \{k, l, p\}

Beam Formers \{k, l\}

RAKEs \{k\}

Decision Logic

\[ K \times L \times P \quad K \times L \quad K \]

[Wang & Poor, T-SP’99]

Optimal, linear, iterative & adaptive versions

Multiuser MIMO Systems
SIMO MUD - Linear Model

*K users*, each transmitting *B symbols*, yields a *linear model*:

\[ y = H b + N(0, \sigma^2H) \]

- \( y \) = \( KB \)-long sufficient statistic vector
- \( b \) = \( KB \)-long vector of symbols
- \( s \) = \( KB \times KB \) matrix of cross-correlations

*Multiuser MIMO Systems*
SIMO MUD - Varieties

• **Optimal**  [Complexity $O(2^{K\Delta})$; $\Delta = \text{delay spread}$]
  - **ML**: $\text{argmax } I(y|b)$
  - **MAP**: $\text{argmax } p(b_{k,i}|y)$

• **Linear**  [Complexity $O((KB)^3)$]
  - **Decorrelator**: $\text{sgn}\{H^{-1}y\}$
  - **MMSE**: $\text{sgn}\{(H+\sigma^2I)^{-1}y\}$

• **Iterative**  [Complexity $O(K\Delta n_{\text{max}})$, etc.]  [Dai & Poor, T-SP’02]
  - **Linear IC**: Gauss-Siedel, Jacobi, conjugate-gradient
  - **Nonlinear IC**: the above with intermediate hard decisions
  - **EM**: symbols are stochastic
  - **Turbo**: symbols are constrained via channel coding

• **Adaptive**  [Sampling, followed by LMS, RLS, subspace, etc.]

\[ y = H b + N(0, \sigma^2 H) \]
**Observation**: MIMO MUD is the same as SIMO MUD, except for the decision algorithm - MI adds further constraints on $b$. 

*Multiuser MIMO Systems*
Space-time Coded Systems

- **Space-time Coded Systems**

- **Single-user Channels**:
  - Encoding of symbols across multiple transmit antennas.

- **Multiuser Channels** [Jayaweera & Poor, EJASP ’02]:
  - **Separation Th’m**: “Full-diversity-achieving single-user ST trellis codes also achieve full diversity in multiuser channels with joint ML detection & decoding (assumes large SNR, quasi-static Rayleigh fading, etc.).”
  - Turbo-style iteration among ST trellis decoding & MUD achieves near-ML performance.
Turbo IC Space-Time MUD (MISO Case)

Multiuser MIMO Systems
Performance \((4 \times 1; \ K=4)\)

Turbo IC-MMSE-ST-MUD Decoder FER Vs. SNR

- Turbo IC-MMSE-ST-MUD: \(\rho=0.75, K=4, X \text{ Ants.}=4\)
- Single User Bound

16-states trellis space-time code

Multiuser MIMO Systems
BLAST-Type Systems

- **BLAST** (Bell Labs Layered Space-Time Architecture)
  - **Basic BLAST** [Foschini, et al.]:
    - Distributes symbols of a single user on multiple tx antennas.
    - Uses MUD to separate different streams using spatial signature.
    - Capacity (Rayleigh) linear in the min\{\#tx,\#rx\} antennae.
    - Capacity gain degrades in interference-limited channels.
  
- **Turbo BLAST** [Dai, Molisch & Poor, *T-WC’04*]:
  - MUD restores the BLAST capacity gain in interference channels.
NETWORK UTILITY IN MU/MIMO SYSTEMS

Joint work with Meshkati, Mandayam & Schwartz.

Multiuser MIMO Systems
Competition in a Multiple-Access Network

- Consider a multiple-access network.
- User terminals are like players in a game, competing for network resources.
- The action of each user affects the performance of others.
- Can model this as a non-cooperative game.
Game Theoretic Framework

**Game:** $G = [\{1, ..., K\}, \{A_k\}, \{u_k\}]$

- $K$: total number of users
- $A_k$: set of strategies for user $k$
- $u_k$: utility function for user $k$

$$u_k = \text{utility} = \frac{\text{throughput}}{\text{transmit power}} = \frac{T_k}{p_k} \left[ \frac{\text{bits}}{\text{Joule}} \right]$$

$$T_k = R_k f(\gamma_k), \text{ where } f(\gamma_k) \text{ is the frame success rate, and } \gamma_k \text{ is the received SIR of user } k.$$
The SIMO Game

- Each user selects its transmit power and uplink (linear) detector to maximize its own utility.
- Nash equilibrium is reached when each user:
  - chooses the MMSE detector as its receiver, and
  - chooses a transmit power that achieves $\gamma^*$, the solution to:

$$ f(\gamma) = \gamma f'(\gamma) $$

Multiuser MIMO Systems
Large-System Analysis

• Consider R-CDMA with spreading gain $N$.
• As $K, N \to \infty$ with $K/N = \alpha$:

$$u_k = \frac{R_k f(\gamma^*)}{\gamma^* \sigma^2} \bar{h}_k \Gamma$$

where

$$\Gamma_{MF} = 1 - \bar{\alpha} \gamma^*$$
for \( \bar{\alpha} < \frac{1}{\gamma^*} \)

$$\Gamma_{DE} = 1 - \alpha$$
for \( \alpha < 1 \)

$$\Gamma_{MMSE} = 1 - \bar{\alpha} \frac{\gamma^*}{1 + \gamma^*}$$
for \( \bar{\alpha} < 1 + \frac{1}{\gamma^*} \)

with

$$\bar{h}_k = \sum_{p=1}^{p} h_{kp}^2$$
and

$$\bar{\alpha} = \frac{\alpha}{P}$$

Two mechanisms:
• power pooling
• interference reduction

Multiuser MIMO Systems
Multiuser detectors achieve higher utility and can accommodate more users compared to matched filter.

Significant performance improvements are achieved when multiple antennas are used compared to single antenna case.

**Multiuser MIMO Systems**
The MIMO Game

• Each user selects its transmit power, uplink (linear) detector, and distribution of power among transmit antennas to maximize its own utility.

• Nash equilibrium is reached when each user:
  – chooses the MMSE detector as its receiver,
  – transmits in the direction of the eigenvector corresponding to the maximal eigenvalue of the “effective” channel matrix (spatial waterfilling), and
  – transmits to achieve SIR $\gamma^\ast$.  

*Multiuser MIMO Systems*
SPECTRAL EFFICIENCY IN MU/MIMO SYSTEMS (BRIEFLY)

Joint work with Husheng Li.
Spectral Efficiency in MU/MIMO Systems

- Consider a system combining DS/CDMA with MIMO.
- Multiple transmit antennas add "diversity" dimensions, but use up "code" dimensions.
- So, there's an overall tradeoff in achieving maximal spectral efficiency (bps/Hz) in such systems.
- For \( K \) equal-rate, equal-power R-CDMA users, can be analyzed in closed form as \( K, N \to \infty \) with \( K/N = \alpha \) fixed.

- **Note:** with \( n_t \) transmit antennas, the "radio" load is \( \alpha n_t \).
Full-Multiplex MIMO (\(\alpha = 0.4 ; n_r = 4\))

- There are diminishing returns for adding antennas when the radio load, \(\alpha n_r\), exceeds 1 (assumes transmitter has no CSI).
- For small SNR, outage issues reverse the advantages of multiple transmit antennas.

\(\text{Multiuser MIMO Systems}\)
Optimal MIMO \( (\alpha = 2.0; n_r = 4) \)

- For large SNR, multiuser efficiency issues reverse the advantages of multiple transmit antennas. (Again there is no tx CSI.)

**Multiuser MIMO Systems**
Summary

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