Spherical Linear Interpolation for Transmit Beamforming in MIMO-OFDM Systems with Limited Feedback

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Outline

- Overview
  - Diversity techniques in MIMO systems
  - Closed-loop transmit beamforming for MIMO-OFDM
- Problem Statement
- Proposed Beamforming Scheme
  - Proposed spherical interpolator
  - Optimization of phase rotation
  - Beamformer quantization + interpolation
- Numerical Simulations and Discussions
Transmit Diversity in MIMO Systems

- No channel state information (CSI) at Transmitter
  - Space-time codes [Tarokh et al. 1998]
- Full CSI at Transmitter
  - Antenna selection [Wittneben et al. 1994], [Heath et al. 1998]
  - Maximum ratio transmission [Lo 1999], [Andersen 2000]
- Partial CSI
  - Adaptive beamforming [Xia et al. 2004]
Covert MIMO to SISO

- Transmit beamforming vector $w$
- Receive combining vector $z$

$y = z^*Hws + z^*n$
Max SNR Solution (MRT/MRC)

- SNR is proportional to \( \frac{|z^* Hw|^2}{|z|} \)

- Max SNR solution is
  \[
  w = \max_{\|w\| = 1} |Hw| \\
  z = Hw
  \]

- Solution is not unique: if \( w \) is optimal, so is \( w e^{j\Theta} \)

- Other solutions possible (constrain \( w \))
  - Selection diversity
  - Equal gain combining / transmission
Beamforming in MIMO-OFDM
Signal Model for One Subcarrier

- Equivalent model for subcarrier $k$
  (identical with a narrowband MIMO case)

$$r(k) = z^H(k)\{H(k)w(k)s(k) + n(k)\}$$
Problem Summary

- Beamforming in MIMO-OFDM requires $\{w(n)\}_{n=0}^{N-1}$
  - Feedback requirements $\propto$ Number of subcarriers

- How can we reduce the number of vectors fed back?
  - Exploit correlation between vectors
  - Send back fraction of vectors
  - Use “smart” interpolation

- How can limit the feedback for each vector?
  - Use quantized beamforming [Love et. al. 2003]
Correlation Between Subcarriers

- **Subchannel correlation**
  \[ \rho_h(d) = \frac{E[|h^H(k+d)h(k)|^2]}{E[||h(k)||^4]} \]
  where \( h(k) = \text{vec}(H(k)) \)

- **Beamformer correlation**
  \[ \rho_w(d) = E[|w^H(k+d)w(k)|^2] \]

- **Simulations**
  - \( \rho_h(d): M_t=4, M_r=1, N=64, K=8, L=\{6,12\} \)
  - \( \rho_w(d): M_t=4, M_r=2, N=64, K=8, L=\{6,12\} \)

subsampling rate number of channel taps
Clustering of Subcarriers

- Clustering ($K=5$)

- Disadvantages
  - Performance degradation in cluster boundary
  - Cluster size (or feedback reduction) is limited
Proposed Beamforming Method

- Subsampling of beamforming vectors

- Reconstruction of beamforming vectors
  - Transmit power constraints force $\|w(k)\| = 1$
  - Spherical interpolation
Conventional Spherical Interpolation

- Spherical averaging [Watson 1983], [Buss et al. 2001]
  \[
  \hat{\mathbf{v}} = \frac{\sum_{i=0}^{p} b_i \mathbf{v}_i}{\| \sum_{i=0}^{p} b_i \mathbf{v}_i \|}
  \]
  - \( p \) is the order, \( \| \mathbf{v} \| = 1 \), \( \sum_{i=0}^{p} b_i = 1 \), and \( b_i \geq 0 \)

- Nonuniqueness of optimal beamforming vectors
  \( \mathbf{w}(k) \) is optimal  \( \iff \)  \( e^{j\theta} \mathbf{w}(k) \) is optimal
  - Random phase rotation \( \theta \) has significant impact on interpolations
  - Performance degradation of spherical interpolators
Proposed Interpolator

■ Spherical Linear Interpolator with phase rotation

\[ \hat{w}(k, \theta_1) = \frac{(1 - c_k)w(1) + c_k\{e^{j\theta_1}w(K + 1)\}}{\| (1 - c_k)w(1) + c_k\{e^{j\theta_1}w(K + 1)\} \|}, \quad 1 \leq k \leq K \]

■ \( c_i = \frac{k-1}{K} \) and \( \theta_1 \) is a parameter for phase rotation.

■ Optimization of phase rotation parameters
  ◆ Maximize the diversity gain
  ◆ Maximize the mutual information
Optimization of Phase Rotation

- Maximizing the minimum channel gain (diversity)

\[ \theta_1 = \arg \max_{\theta \in [0, 2\pi]} \min \{ \| H(k) \hat{w}(k, \theta) \|^2, \ 1 \leq k \leq K \} \]

- Difficult to get a closed-form solution

- Grid search

\[ \theta_1 = \arg \max_{\theta \in \Theta} \min \{ \| H(k) \hat{w}(k, \theta) \|^2, \ 1 \leq k \leq K \} \]

- \( \Theta = \{ 0, \frac{2\pi}{P}, \frac{4\pi}{P}, \ldots, \frac{2(P-1)\pi}{P} \} \) and \( P \) is the number of quantized levels
Alternative Solution

Observation
- Subcarrier \((K/2+1)\) suffers from the largest distortion
- Subcarrier \((K/2+1)\) has the worst average channel gain

Approximation of cost function

\[
\theta_1 = \arg \max_{\theta \in [0, 2\pi)} \|H(K/2 + 1)\hat{w}(K/2 + 1, \theta)\|^2
\]

\[
= \arg \max_{\theta \in [0, 2\pi)} \|H(K/2 + 1)\{w(1) + e^{j\theta}w(K + 1)\}\|/w(1) + e^{j\theta}w(K + 1)
\]
Closed-Form Solution

- Differentiation with respect to $\theta$
  \[
  \text{Imag}(\alpha_1 e^{j\theta} + \alpha_2) = 0
  \]
  - $\alpha_1$ and $\alpha_2$ are complex constants satisfying $|\alpha_1| \geq |\text{Imag}(\alpha_2)|$
  - Always, there exist two solutions.

- Second derivative
  \[
  \frac{d}{d\theta} \text{Imag}(\alpha_1 e^{j\theta} + \alpha_2) = -2\text{Real}(\alpha_1 e^{j\theta})
  \]
  - The solution making the second derivative be negative is the optimal solution.  
    \[\text{closed-form solution} \]
Optimization in Terms of Capacity

- Maximizing the capacity

\[ \theta_1 = \arg \max_{\theta \in [0, 2\pi)} \sum_{k=1}^{K} \log_2 \left( 1 + \frac{\|H(k)\hat{w}(k, \theta)\|^2}{N_0} \right) \]

- Grid search

\[ \theta_1 = \arg \max_{\theta \in \Theta} \sum_{k=1}^{K} \log_2 \left( 1 + \frac{\|H(k)\hat{w}(k, \theta)\|^2}{N_0} \right) \]

\[ \Theta = \{0, \frac{2\pi}{P}, \frac{4\pi}{P}, \ldots, \frac{2(P-1)\pi}{P}\} \]

- Closed-form solution is the same as in diversity case
Simulation Environments

- **MIMO channels**
  - Each channel impulse response has 6 taps ($L=6$) with uniform profile and follows i.i.d. $CN(0,1/L)$.
  - The channels between different transmit and receiver antennas pairs are independent.

- **Noise**: i.i.d. with $CN(0,N_0)$

- **Simulation parameters**
  - $M_t=4$ (# of Tx antennas), $M_r=2$ (# of Rx antennas)
  - $N=64$ (# of subcarriers), $K=8$ (subsampling rate), $L=6$

- **Basic assumptions**
  - MRC at the receiver
  - No delay and no transmission error in the feedback channel
  - QPSK, no water-filling
Simulation Results – Channel Gain

- OFDM subcarrier vs. effective channel gain
Simulation Results – Uncoded BER

- No channel coding
Simulation Results – Capacity

- No power control for diversity techniques
Simulation with Quantization

- Quantization of beamforming vectors
  - By using the codebook in [Love & Heath 2003]
  - 6 bits of feedback for each beamforming vector
- Phase optimization
  - Grid search with $P=4$ (2 bits of feedback per phase)
- Feedback requirements
  - Ideal: $6N = 384$ bits / OFDM symbol
  - Clustering: $6N/K = 48$ bits / OFDM symbol
  - Selection: $2N = 128$ bits / OFDM symbol
  - Proposed: $8N/K = 64$ bits / OFDM symbol
  - Orthogonal STBC (OSTBC): none
Simulation Results - Channel Gain

- $M_t=4$, $M_r=2$, $N=64$, $K=8$, $L=6$
Simulation Results - BER

- Selection
  - Full diversity order
  - Array gain loss

- Proposed
  - Diversity order loss
  - Additional array gain
Simulation Results - BER

- BER degradation by channel prediction error

- Prediction error model

\[ \tilde{H}(k) = \gamma H(k) + \sqrt{1 - \gamma^2} U(k) \]

where

\( U(k) \) is a prediction error
Simulation Results - Capacity

- No power control
Simulation Results – Coded BER

- 1/2 convolutional code
- Interleaving/deinterleaving
- Frame length
  - 30 OFDM symbols
- 64 bits / OFDM symbol

![Graph showing BER vs. Eb/N0 (dB)]
Conclusions

Summary
- Interpolation for transmit beamforming in MIMO-OFDM proposed
- Relates to problem of spherical quantization

Future work
- Proposed system does not obtain MIMO capacity
- Develop “interpolated” waterfilling solutions