ABSTRACT
We consider the problem of passive localization of a low-grazing-angle source employing polarization sensitive sensor arrays. We present a general polarimetric signal model that takes into account the direct field and the multipath interference produced by reflections from smooth and rough surfaces. Applying the Cramér-Rao bound (CRB) and mean-square angular error (MSAE) bound, we analyze the performance of different array configurations, which include an electromagnetic vector sensor (EMVS), a distributed electromagnetic component array (DEMCA), and a distributed electric dipole array (DEDA). By computing these bounds, we show significant advantages in using the proposed diversely polarized arrays compared with the conventional scalar-sensor arrays.

1. INTRODUCTION
The presence of a signal source and a receiver near a reflecting surface is a common scenario in many radar and communication systems; for instance, low-angle radar tracking systems [1] and terrestrial radio links [2]. Under this condition, the received signal can be modeled as the sum of fields arriving through the direct path and the scattered from the surface. A major challenge in multipath propagation at a low-grazing-angle is that the angular separation between direct and reflected paths is smaller than the beamwidth of the receiver antenna array; hence, the reflected signal cannot be spatially filtered. Most of the previous work in this area has considered scalar-sensor arrays [3]-[6] which cannot resolve very close source arrival paths [7] nor can it provide a good estimation of position parameters: direction-of-arrival (DOA), range, and altitude.

We propose to use diversely polarized sensor arrays, which measure more than one electric and/or magnetic component of the field, to overcome the above difficulties and improve the estimation of all the source position parameters. In addition, diversely polarized arrays can be applied to estimate the polarization parameters of the received signal. In this paper, we present a general polarimetric signal model that takes into account the interference of the direct field with the field reflected by smooth and rough surfaces. Then, using the Cramér-Rao bound (CRB) and the mean-square angular error bound (MSAE), we analyze the performance of different array configurations, including an electromagnetic vector sensor (EMVS) [8], a distributed electromagnetic component array (DEMCA) [9], [10]; and a distributed electric dipole array (DEDA). We show that the proposed arrays significantly reduce the parameter error estimation compared with the scalar array.

This paper is organized as follows: In Section II, we describe a general measurement model for sensor arrays of any type of polarization. In Section III, we define the CRB and MSAE as our measures of system performance. In Section IV, the proposed polarimetric arrays are analyzed via computer simulations. Conclusions are given in Section V.

2. PROBLEM DESCRIPTION AND MODELING
In this section, we first present the measurement model for a smooth surface; then, we extend it for a rough surface. Next, we discuss our statistical assumptions on the signal, multipath interference, and noise.

2.1. Signal Model for Smooth Surfaces
Consider a receiver and a signal source at heights \( h_r \) and \( h_s \) above a flat smooth surface, separated by a distance \( r \) on the ground, as shown in Figure 1. We choose a Cartesian coordinate system such that the receiver is located at the origin \( O \). The source elevation angle \( \theta \) and the grazing angle \( \psi \) of the reflected signal are measured from the horizontal plane,
and the azimuth angle $\phi$ from the $x$-axis. It can be easily shown that angle and length parameters are related by

$$\tan \vartheta = \frac{h_s - h_r}{r}, \quad \tan \psi = \frac{h_s + h_r}{r}. \quad (1)$$

These geometrical relationships show that the source location, i.e. range $r$ and altitude $h_s$, can be determined from the bearing angles, assuming the receiver position is known.

We assume the propagating field is a transverse electromagnetic (EM) wave. To describe the direct wave, we define a right-handed orthonormal triad $(k, h, v)$, where:

$$k = [\cos \phi \cos \vartheta, \sin \phi \cos \vartheta, \sin \theta]^T,$$
$$h = [-\sin \phi, \cos \phi, 0]^T,$$
$$v = [-\cos \phi \sin \vartheta, -\sin \phi \sin \vartheta, \cos \vartheta]^T. \quad (2)$$

The vector $k$ is pointing from the receiver toward the source; vectors $h$ and $v$ span the plane where the electric and magnetic field vectors lie. Replacing $\vartheta$ by $\psi$ in (2), we can also define a coordinate system for the reflected wave.

At an observation point $r$, the complex envelope of the direct electric vector $E_d(t)$ is given by [8]

$$E_d(t) = [h, v]p_s(t)e^{j2\pi r^T k/\lambda} \quad (3)$$

where the exponential term represents the phase of the plane-wave at position $r$, $\lambda$ is the transmitted signal wavelength, and $s(t)$ is the complex envelop of the signal. The polarization vector $p$ is defined as

$$p = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta \\ j \sin \beta \end{bmatrix} \quad (4)$$

where the angles $\beta$ and $\alpha$ are the ellipticity and orientation of the polarization ellipse depicted by the electric field vector in the plane spanned by $h$ and $v$. In a plane wave, the electric and magnetic fields are orthogonal to each other and to the direction of propagation. Hence, within a normalization factor, the complex envelope of the direct magnetic field vector at $r$ is given by

$$H_d(t) = k \times E_d(t) = [v, -h]p_s(t)e^{j2\pi r^T k/\lambda} \quad (5)$$

where $\times$ is the cross-product operator. The components of the direct electric and magnetic field can be stacked, forming a 6-element complex vector:

$$\zeta_d(t) = \begin{bmatrix} E_d \\ H_d \end{bmatrix} = g(\phi, \vartheta)V(\phi, \vartheta)p_s(t). \quad (6)$$

where we have defined

$$g(\phi, \vartheta) = e^{j2\pi r^T k/\lambda}, \quad (7)$$
$$V(\phi, \vartheta) = \begin{bmatrix} h & v \\ v & -h \end{bmatrix}. \quad (8)$$

Similarly, the components of the EM field reflected from the smooth surface can also be arranged in a vector. However, the reflected wave experiences a change of phase and amplitude with respect to the direct signal:

$$\zeta_r(t) = e^{j\delta}g(\phi, \psi)V(\phi, \psi)\Gamma_0 p_s(t - \tau) \quad (9)$$

where $\delta = 2\pi \Delta r/\lambda$ is the phase shift due to the length difference $\Delta r$ between the two paths. For low-grazing angle propagation ($r \gg h_r, h_s$), the path length difference is approximated by $\Delta r \approx 2h_r h_s/r$. The time delay is given by $\tau = \Delta r/c$, where $c$ is the propagation velocity. The complex reflection matrix is $\Gamma_0 = \text{diag}(\gamma_h, \gamma_v)$; expressions of Fresnel reflection coefficients $\gamma$ can be found in classical textbooks of EM. The scalar $g$ and the matrix $V$ are defined as in (7) and (8), replacing $\vartheta$ by $\psi$.

Assuming narrowband condition, we can approximate $s(t) \simeq s(t - \tau)$. Then, the total field over the surface is the superposition of the direct and reflected field components:

$$\zeta(t) = [g(\phi, \vartheta)V(\phi, \vartheta) + e^{j\delta}g(\phi, \psi)V(\phi, \psi)\Gamma_0]p_s(t) \quad (10)$$

### 2.2. Measurement Models

Consider an array of electric and magnetic sensors with different polarizations. We assume that the array size is much smaller than the source-receiver distance; hence, the incoming wave is a plane wave with the same direction of arrival at all the sensors. In the presence of additive noise $e(t)$, the output of an array of $m$ sensors is

$$y(t) = a_0(\theta)s(t) + e(t), \quad t = 1, \ldots, N \quad (11)$$

where $y(t) \in C_{m \times 1}$ is the measurement vector, $a_0(\theta) \in C_{m \times 1}$ is the response vector of the sensor array, and $\theta = \ldots$
\([\phi, \theta, \psi, \alpha, \beta]^T\) is the vector of the unknown parameters of interest. The response of the \(l\)th sensor is given by

\[
a_{l}(\theta) = [g_l(\phi, \theta) v_l V(\phi, \theta) + e^{j \zeta_l} g_l(\phi, \psi) v_l V(\phi, \psi) \Gamma_0] p
\]

(12)

for \(l = 1, \ldots, m\), where \(g_l\), defined in (7), is the phase shift due to the sensor position \(r_l\), and \(v_l\) is a \(1 \times 1\) vector of 1 and 0 entries selecting the component of the EM field, given by (10), which is being measured by the \(l\)th sensor. For example, the selection vector is \(v = [0, 0, 1, 0, 0, 0]\) for a dipole parallel to the \(z\)-axis. Stacking the response of each sensor and arranging the sensor phase shifts in a diagonal matrix \(G = \text{diag}(g_1, \ldots, g_m)\), the array response can be written as

\[
a_{0}(\theta) = \begin{bmatrix} G(\phi, \theta) \Upsilon V(\phi, \theta) + e^{j \zeta} G(\phi, \psi) \Upsilon V(\phi, \psi) \Gamma_0 \end{bmatrix} p
\]

(13)

where \(\Upsilon = [v_1^T, \ldots, v_m^T]^T\) (see [9], [10] regarding this notation). Note that equation (13) is a general expression for any array, which could be formed by scalar or diversely polarized sensors.

2.3. Rough Surfaces

When the surface is smooth, the reflected signal is totally coherent with the direct signal; for rough surfaces, the reflected signal consists of a coherent component with reduced magnitude and a diffuse component [1]. Then, the specular component for rough surfaces is represented as in (9); however, the reflection matrix is replaced by [1]

\[
\Gamma = \Gamma_0 e^{-8(\pi \sigma_h \sin \psi / \lambda)^2}
\]

(14)

where \(\sigma_h\) is the standard deviation of the distribution of the surface heights.

The diffuse term accounts for the field scattered by the irregularities on the surface. We assume that this field is the result of the contribution of many independent point scatterers. As a consequence of the central limit theorem, the diffuse component can be modeled as a Gaussian random process with zero mean [11], [12]. We also assume that these point scatterers are randomly located. If the position distribution of the scatterers is symmetric around the specular reflection point, then the DOA of the diffuse component is concentrated in the direction of the specular component. This assumption is not only intuitive for an homogeneous surface but is also supported by experimental data analyzed in References [12]-[14].

The previous representation of the reflected wave does not provide a clear interpretation about its polarization state. To correct this deficiency, we apply the following decomposition lemma [15].

Lemma: Let \(\xi(t)\) be a \(2 \times 1\) complex vector whose entries are the horizontal and vertical components of a plane-wave. Its covariance \(P_\xi \in \mathbb{C}^{2 \times 2}\) can be decomposed as

\[
P_\xi = \sigma_p^2 p_p^e p_e^* + \sigma_u^2 I_2
\]

(15)

where either \(\sigma_p^2\) or \(\sigma_u^2\) can be zero. \(I_2\) is the \(2 \times 2\) identity matrix. \(^*\) is the conjugate transpose, and \(p_p\) is the polarization vector defined as in (4) in terms of the angles \(\alpha_\xi \in (-\pi/2, \pi/2)\) and \(\beta_\xi \in [-\pi/4, \pi/4]\). Furthermore, assuming \(\sigma_u^2 > 0\), this decomposition is unique if and only if \(|\beta_\xi| \neq \pi/4\). The detailed proof is given in [16].

The lemma states that a plane wave \(\xi(t)\) can be divided into polarized and unpolarized components with powers \(\sigma_p^2\) and \(\sigma_u^2\), respectively. Applying it to the reflected wave, we can relate the polarization component to the coherent term, where the polarization vector and power are given by \(p_p = \Gamma p\) and \(\sigma_p^2 = \gamma_p^2 = E|s(t)|^2\) for any \(t\). The unpolarized component is associated with the diffuse part of the plane wave, which we denote \(u(t)\) in \(\mathbb{C}^{2 \times 1}\). It follows from (15) that the horizontal and vertical components of the diffuse field are uncorrelated (the same assertion is deduced from the analysis of real data in [13]). Hence, to consider the reflections from a rough surface, the EM field vector given by (9) is extended as follows:

\[
\xi(t) = g(\phi, \psi) V(\phi, \psi) \left[e^{j \zeta} \Gamma p s(t) + u(t)\right].
\]

(16)

Then, combining (6) and (16), the total EM field vector is given by

\[
\xi(t) = \begin{bmatrix} g(\phi, \theta) V(\phi, \theta) + e^{j \zeta} g(\phi, \psi) V(\phi, \psi) \Gamma \end{bmatrix} p s(t) + g(\phi, \psi) V(\phi, \psi) u(t).
\]

(17)

Despite the fact that the diffuse component carries no useful message, it provides information about the source position through its bearing angles. Hence, this component is considered as “signal” in the measurement model, instead of as part of the additive noise. Then, the output of an array of \(m\) diversely polarized sensors is given by

\[
y(t) = A(\theta) x(t) + e(t), \quad t = 1, \ldots, N,
\]

(18)

where

\[
x(t) = \begin{bmatrix} s(t), u^T(t) \end{bmatrix}^T,
\]

\[
A(\theta) = \begin{bmatrix} a(\theta), G(\phi, \psi) V(\phi, \psi) \end{bmatrix},
\]

\(A(\theta) \in \mathbb{C}^{m \times 3}\) is the array response matrix, and \(a(\theta)\) is defined as in (13), using the reflection matrix for a rough surface given in (14).

2.4. Statistical Assumptions

We assume that the signals \(s(t)\) and \(u(t)\), and the noise \(e(t)\) are independent identically distributed (i.i.d.) complex circular Gaussian processes with zero mean. In addition, we
assume that the signals and noise are independent random processes. These processes are completely characterized by their covariances, which are respectively \( \sigma^2_s, \sigma^2_u, \sigma^2_f \), and \( \sigma^2 I_m \). For simplicity, we assume the same noise variance at each sensor; however, the results can be extended for different covariance structures [8]. Under these assumptions, the output of the array is also an i.i.d. zero-mean complex Gaussian process with covariance matrix

\[
C_y(\eta) = A(\theta)C_x A^*(\theta) + \sigma^2 I_m,
\]

(19)

where \( C_x = \text{diag}(\sigma^2_s, \sigma^2_u, \sigma^2_f) \) and \( \eta = [\theta^T, \sigma^2_s, \sigma^2_u, \sigma^2_f]^T \) is the vector of unknown parameters of the model. The entries of the vector \( \theta \) are the parameters of interest, and the powers \( \sigma^2_s, \sigma^2_u, \) and \( \sigma^2_f \) are considered nuisance parameters.

3. MEASURES OF PERFORMANCE

The Cramér-Rao bound (CRB) is a universal lower bound on the variance of all unbiased estimators of a set of parameters. It is defined as the inverse of the Fisher information matrix (FIM):

\[
\text{CRB}^{-1}(\eta) = \text{FIM}(\eta) = -E\left[ \frac{\partial^2 \ln p(y; \eta)}{\partial \eta \partial \eta^T} \right]
\]

(20)

If the data have an i.i.d. zero-mean complex Gaussian distribution, the \((i, j)\)th entry of (20) can be written as [17]

\[
[FIM(\eta)]_{ij} = N \text{tr} \left[ C_y^{-1} \frac{\partial C_y}{\partial \eta_i} C_y^{-1} \frac{\partial C_y}{\partial \eta_j} \right]
\]

(21)

where \( N \) is the number of snapshots, \( C_y \) is the data covariance matrix given by (19), and \( \text{tr} \) indicates the matrix trace operator. When the desired parameters are a function of the original parameters, i.e. \( \nu = g(\eta) \), the CRB is [17]

\[
\text{CRB}(\nu) = \frac{\partial g(\eta)}{\partial \eta} \text{FIM}^{-1}(\eta) \frac{\partial g(\eta)}{\partial \eta}^T.
\]

(22)

If \( \nu = [r, h_s]^T \), the function \( g \) gives the relation between the length and angle parameters, which can be derived from (1).

Estimating the source azimuth and elevation angles is equivalent to estimating the bearing vector \( k \). The angular difference between \( k \) and its estimate is called angular error of the direction estimator. A natural measure of the estimator performance is given by the mean-square angular error (MSAE). In [8], it is shown that the MSAE lower bound (MSAE_B) of any unbiased estimator of \( k \) is

\[
\text{MSAE}_B(\phi, \theta) = \cos^2 \theta \text{ CRB}(\phi) + \text{CRB}(\theta)
\]

(23)

This bound can be considered as an overall measure of error for the source DOA estimation (or reflected DOA, replacing \( \theta \) by \( \psi \)).

Both bounds, the CRB and the MSAE_B, are independent of the estimation algorithm, providing a measure of potential performance attainable by the system.

In this section, we analyze the performance of source localization for the following sensor arrays: (a) EMVS [8]; (b) DEMCA [9], [10]; (c) DEDA; and (d) a scalar array. The EMVS consists of 6 co-located sensors, each measuring one component of the EM field. For a fair comparison, the other three arrays have the same number of sensors, \( m = 6 \); however, the sensors are located at different positions, forming a uniform circular array parallel to the horizontal plane, as depicted in Figure 2. The DEMCA has one sensor for each component of the EM field, as well. The DEDA measures the three components of the electric field, and the scalar array measures only one component of the EM field.

We carried out computer examples to study the performance of the former arrays. The range between the source and receiver is \( r = 10,000\lambda \), the receiver height is \( h_r = 60\lambda \), and the receiver height \( h_s \) varies from 0 to 200\lambda (\( \lambda = 0.3m \)). Under these conditions, the angular separation between the direct and reflected wave is \( 0.7^\circ \). The inter-element distance is \( d = 0.5\lambda \). We consider a linear polarized field (\( \beta = 0^\circ \)) with the two most significant orientations: the electric field is horizontally and vertically polarized, i.e., \( \alpha = 0^\circ \) and \( \alpha = 90^\circ \), respectively. We assume the reflections are produced on a seawater surface whose relative complex permittivity is \( \epsilon_r \approx 80 - j240\lambda \), under calm (\( \sigma_h = 0m \)) and rough (\( \sigma_h = 2m \)) sea state conditions. The signal to noise ratio is \( \text{SNR} = 10 \log_{10}(\sigma^2_s/\sigma^2_n) = 10\text{dB} \).

For a rough sea state, the power ratio between the signal and diffuse multipath component is \( 10 \log_{10}(\sigma^2_s/\sigma^2_n) = 7\text{dB} \). (Note: this value was selected to approximately match the
measurements reported in [13].

An array of sensors measuring only one component of the EM field is not capable of estimating the signal polarization. To compare the scalar array with the other arrays, we assume that the polarization angles $\alpha$ and $\beta$ are known.

Figure 3 gives the square root of the $\text{MSAE}_B$ of the source DOA as a function of the source height $h_s$ and the signal polarization when the reflecting surface is calm seawater. The peaks in the bounds occur when there is strong signal fading produced by the interference of the direct and reflected fields. It is shown that the DEMCA performs somewhat better (approximately 1dB) than EMVS because the former exploits the information provided by the differential phase between sensors. The performance difference between these arrays can be increased by enlarging the interelement distance $d$. In addition, Figure 3 shows that EMVS and DEMCA perform approximately 15dB better than the scalar array. This difference implies that exploiting the polarization aspect of the signal produces an improvement in the estimation of the unknown parameters. The performance of DEDA highly depends on the polarization of the signal as well as on the source azimuth angle $\phi$, as shown in Figure 4. We can state that EMVS and DEMCA are more robust than DEDA, showing more a stable performance with respect to variations in polarization and azimuth angle of the source.

Similar remarks can be obtained computing $\text{MSAE}_B$ of the reflected signal DOA and range CRB. Because of length constraints, those graphs are not shown in the paper.

Simulations for rough seawater were also performed (see Figure 5 which shows the $\text{MSAE}_B$ for the source DOA as a function of the source height $h_s$ when the source electric field is horizontally polarized). Results on the bound for EMVS, DEMCA, and DEDA are similar to the smooth surface condition. However, the CRB for the proposed scalar array does not exist, since the Fisher information matrix is singular. The absence of the lower bound means that the depicted scalar array cannot be used in combination with the proposed model for rough surfaces, since the reflected wave is represented by two signals with the same bearing and different polarization state. This fact indicates another relevant advantage of diverse polarized arrays.

5. CONCLUSIONS

We have addressed the problem of passive source direction of arrival and range estimation using polarimetric sensor arrays located near a reflecting surface. We have presented a general measurement model for receive arrays composed of sensors with diverse polarization. In this new polarimetric model, we have proposed to decompose the multipath component into polarized and unpolarized terms, following the decomposition lemma which we have stated in Section 2.3. We have used the Cramér-Rao bound and the mean-square

Figure 3: Square root of $\text{MSAE}_B$ for the estimated source direction of arrival versus the source height $h_s$ over calm seawater (azimuth angle $\phi = 45^\circ$): (a) horizontally and (b) vertically polarized source field.

Figure 4: Square root of $\text{MSAE}_B$ for the estimated source direction of arrival versus the source azimuth angle $\phi$ over calm seawater (source height $h_s = 50\lambda$).
angular error bound as performance measures for studying
polarimetric arrays. In addition, we have analyzed and com-
pared different arrays by computing the former bounds for
the source range and direction of arrival estimation. We
have shown that it is possible to significantly reduce the er-
ror estimation of the unknown parameters when the full EM
information is exploited using EMVS or DEMCA. Further-
more, we have shown that the performances of EMVS and
DEMCA are more stable than that of DEDA, since they are
independent of the polarization state and azimuth angle of
the source.

6. REFERENCES


arrival on a terrestrial microwave link,” IEEE Trans.

monopulse radar tracking errors,” IEEE J. Ocean. Eng.,

[4] T. Lo and J. Litva, “Use of a highly deterministic multi-
path signal model in low-angle tracking,” IEEE Proc. F.,

multipath dominant environments, Part I: known multi-

low angle radar tracking in multipath,” IEEE National

with an array of antennas having diverse polarizations,”
IEEE Trans. Antennas Propagat., vol. 31, pp. 231-236,


with distributed electromagnetic component sensor ar-
ray processing,” Int. Symp. on Signal Processing and Its

partially calibrated distributed electromagnetic com-
ponent sensor array,” Workshop on Statistical Signal

vector model of microwave reflection from the ocean,”
1956.

[12] A. Straiton and C. Tolbert, “Measurement and analy-
sis of instantaneous radio height-gain curves at 8.6 mil-
limeters over rough surfaces,” IEEE Trans. Antennas

[13] C. Beard, “Coherent and incoherent scattering of mi-
crowaves from the ocean,” IEEE Trans. Antennas Prop-

[14] T. Lo and J. Litva, “Characteristics of diffuse multi-
path at low grazing angles in naval environments,” Int.
Geoscience and Remote Sensing Symp., IGARSS ’91,

[15] B. Hochwald and A. Nehorai, “Polarimetric model-
ing and parameter estimation with applications to re-
 mote sensing,” IEEE Trans. Signal Process., vol. 43,

Ph. D. dissertation, Yale University, New Haven, CT,

Processing: Estimation Theory, Englewood Cliffs, NJ: