Super-resolution source localization through data-adaptive regularization


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Motivation

SensorWeb project

- Data fusion in large arrays of microsensors
- Dynamic and uncertain environment
- Highly irregular stochastic array geometry
The goals of our research

• To exploit the spatial sparsity of the signal field to achieve robust high resolution estimates of the location parameters of the sources.

• To explore the regularized linear inverse problem framework used successfully in other fields (e.g. image restoration) as an approach for source localization

• To get an improved performance compared to standard DOA methods:
  – Higher resolution
  – Lower sidelobes
  – Robustness to noise levels and signal coherence
  – Better performance with short data records
  – Self calibration or robustness to model uncertainties
Currently Existing Source Localization Methods and their shortcomings

• Non-parametric methods
  – Conventional beamforming
    *Low resolution*
  – Capon’s minimum variance method
    *Fails to work with a poor covariance matrix estimate*
  – Subspace-based methods such as MUSIC
    *High sensitivity to model errors, and signal coherence*

• Parametric methods
  – Deterministic and stochastic maximum likelihood
    *Requires very precise initial guess for convergence*
Best basis selection and sparsity

• Motivation
  – Good resolution requires sparse representation
    (sparsity means small number of non-zero coefficients)
  – The minimum spanning basis can represent sparsely only
    a small subset of possible signals.
  – Overcomplete basis, where the vectors are properly chosen,
    can greatly extend the number of sparsely representable
    signals.
  – However, due to overcompleteness the representation of a
    signal is not unique, and the sparse one has to be searched
    for among the possibilities.
Search for sparse solutions representation

- $\mathbf{y}$ - data vector, $\mathbf{A}$ matrix with the overcomplete basis vectors as its columns, $\mathbf{x}$ the representation of $\mathbf{y}$ by $\mathbf{A}$. i.e. $\|\mathbf{Ax} - \mathbf{y}\|_2^2$ and $\mathbf{x}$ is sparse.

- Methods to enforce sparsity
  - combinatorial optimization
  - Matching pursuit (a greedy algorithm sequentially reducing the residual) and its variants
  - sparsity functional minimization - optimization framework
    $J(\mathbf{x}) = J_1(\mathbf{x}, \mathbf{y}) + \alpha J_2(\mathbf{x})$, $J_1$ - data fidelity, $J_2$ - prior.

- Another interpretation: the problem is $\mathbf{Ax} + \mathbf{n} = \mathbf{y}$ ill-posed and the sparsity constraint serves as regularization, i.e. it provides the necessary prior information about $\mathbf{y}$. 
Measures of sparsity for $J_2(x)$

- number of nonzero elements (not continuous)
- Entropy-like $H(x) = - \sum_{i=1}^{N} x_i \times \log(x_i)$
- $L_1$ and $L_p$ norms, $0 < p < 1$, $\|x\|_p^p = \sum_{i=1}^{N} (x_i^p)$.
**Observation Model**

- M sensors, K source signals $u_k(t), k \in \{1, ..., K\}$, sensor outputs $g_m(t)$ and noise $n_m(t)$.

$$g_m(t) = \sum_{k=1}^{K} u_k(t - \tau_m(\theta_k)) + n_m(t)$$

$\tau_m(\cdot)$: time delay to the $m$-th sensor, $\theta_k$ - direction of arrival of $k$-th signal

- In frequency domain (combining data from all sensors):

$$g(\omega) = A(\omega, \Theta)u(\omega) + n(\omega)$$

where $A_{mk}(\omega, \Theta) = \exp(-j\omega\tau_m(\theta_k))$

- Linear relationship in time domain in narrowband case:

$$g(t) = A(\Theta)u(t) + n(t)$$

- Note $A(\Theta)$ depends on actual source DOAs.
Source Localization observation Model

- Let \( \{ \beta_1, ..., \beta_{N_\beta} \} \) be a sampling grid of all directions of arrival.
- Define a \( N_\beta \times 1 \) vector \( s(t) \), the \( i \)-th element of \( s(t) \) satisfies

\[
s_i(t) = \begin{cases} 
  u_k(t), & \text{if } \beta_i = \theta_k \\
  0, & \text{otherwise}
\end{cases}
\]

- Define a \( M \times N_\beta \) steering matrix \( A \) (linking all potential DOAs to all sensors), \( A \) is independent of \( \theta_k \)
- Resulting “overcomplete” observation model:

\[
g(t) = As(t) + n(t)
\]

- Determine DOAs from peaks in reconstructed signal energy
Translating DOA estimation problem into the sparse representation problem

- Version 1: Produce the signal estimate at each time point
  \[ J(s(t)) = \| g(t) - As(t) \|^2_2 + \alpha \| s(t) \|^p_p \]
  - Computational overkill
  - May not be robust to high noise levels

- Version 2: Combine temporal data prior to processing
  - Complex amplitudes of source signals need to be non-zero-mean.

- Version 3: Frequency representation, narrowband signals
  \[ J(s(\omega_0)) = \| g(\omega_0) - A(\omega_0)s(\omega_0) \|^2_2 + \alpha \| s(\omega_0) \|^p_p \]
Solution of the Optimization Problem

- Continuous approximation to the cost function:

\[ J_\varepsilon(s(\omega_0)) = \| g(\omega_0) - As(\omega_0) \|_2^2 + \alpha \sum_{i=1}^{N_\beta} (|s_i(\omega_0)|^2 + \varepsilon)^{p/2} \]

- Gradient of the cost function:

\[ \nabla J_\varepsilon(s(\omega_0)) = H(s(\omega_0)) s(\omega_0) - A^H g(\omega_0) \]
Solution of the Optimization Problem

- Iterative scheme

\[ \mathbf{H} \left( \hat{s}^{(n)} \right) \hat{s}^{(n+1)} = \mathbf{A}^H \mathbf{g} \]

where \( n \) denotes the iteration number, and:

\[ \mathbf{H}(s) \triangleq \mathbf{A}^H \mathbf{A} + \alpha \Lambda(s) \]

\[ \Lambda(s) \triangleq \text{diag} \left\{ \frac{p/2}{(|s_i|^2 + \epsilon)^{1-p/2}} \right\} \]

- Can be interpreted as a Quasi-Newton method with Hessian approximation \( \mathbf{H}(\cdot) \) and unit step-size.

- Each step of the algorithm solves a quadratic optimization problem.
Simulation Setup

- Uniform linear array with $M = 8$ sensors
- Two narrowband signals in the far-field
- Total number of snapshots (time samples) $T = 200$
- Use $p = 0.1$ in our objective function
- Choose $\alpha$ by subjective assessment
Results with uncorrelated sources

$\theta_s: 50^\circ, 120^\circ, \text{SNR} = 10 \text{ dB}$

$\theta_s: 50^\circ, 60^\circ, \text{SNR} = 20 \text{ dB}$

$\theta_s: 50^\circ, 60^\circ, \text{SNR} = 10 \text{ dB}$

$\theta_s: 50^\circ, 60^\circ, \text{SNR} = 5 \text{ dB}$
Results with coherent sources

- DOAs: $50^\circ$ and $60^\circ$.
- SNR = 20 dB.
Progress of the reconstruction with iterations

- Iteration 1
- Iteration 2
- Iteration 3
- Iteration 4
- Iteration 5
- Iteration 6
- Iteration 7
- Iteration 8
- Iteration 13
Characterization of Performance as a function of SNR

- DOAs: 50° and 65°.
- Number of independent trials = 200.
Characterization of Performance as a function of the number of snapshots

- DOAs: 50° and 65°. SNR = 10 dB.
- Number of independent trials = 200.
Issues with the current approach

- Choosing the regularization parameter
  - $\alpha$ is currently chosen based on subjective assessment
  - depends on the number and energy of the signals, and noise levels
  - An automatic selection method is under consideration

- Asymptotic bias for closely spaced signals
  - For closely spaced DOA’s, the reconstruction exhibits bias
  - Bias does not disappear with more snapshots
  - Bias and resolution can be traded off by adjusting $p$
Asymptotic Bias (Detection vs. SNR)

- DOAs: 50° and 60°.
- Number of independent trials = 200.
Extension: Nearfield case

- The steering matrix is parametrized by range and bearing
- The 2-d search grid is flattened into a vector
- The same reconstruction procedure is applied

Conventional beamforming results

Lp regularization results
Extension: Wideband case

- In frequency domain:
  \[ g(\omega) = A(\omega, \Theta)u(\omega) + n(\omega) \]
  where \( A_{mk}(\omega, \Theta) = \exp(-j\omega \tau_m(\theta_k)) \)

- Two possible approaches:
  - Frequency by frequency reconstruction
  - Aggregating the model for all frequencies and inverting at once
Wideband case, frequency by frequency reconstruction

- For small number of snapshots DFT sidelobes cause power leakage and frequency mismatch

Conventional beamforming results  Lp regularization results
Wideband case, full model inversion

- Directly appropriate for multiple harmonics only
- Full model inversion forces sparsity in frequency as well as in space
- Can be extended to more general wide-band signals by using a different prior for frequency, e.g. smoothness

Conventional beamforming results  Lp regularization results
Microphone array experiments

- Reverberations due to the walls rendered the DOA problem less tractable
- Additional difficulties include high levels of spatially and temporally non-stationary noise
Extension to self calibration and model-error robustness

- Introduce the sensor position error vector $\xi$

\[ g = A(\xi)s + n \]

- A possible path: Change the optimization cost function to

\[ J(s) = J_1(s, \xi) + \alpha J_2(s) + \beta \| \xi \|^2 \]

- Iterative two-step optimization procedure:
  - Hold $\xi$ constant, solve for $s$
  - Hold $s$ constant solve for $\xi$, and repeat until convergence.

- Still under development
Summary and conclusions

- Sparsity is a valuable asset
- Feature-preserving regularized Linear Inverse Problem framework is a viable and versatile approach to source localization
- An iterative algorithm for the minimization of non-convex cost function
- Simulation suggests that the method outperforms conventional methods in several scenarios
- Many challenges remain
Current and Future Work

- Issues to investigate about the current approach:
  - Shifts in locations of closely spaced peaks
  - Automatic choice of parameters
  - Relation to existing methods

- Extensions:
  - Non-linear array configurations
  - General broadband signals
  - Directional sensors
  - Attenuating, dispersive medium
  - Sensor location uncertainties