MIMO Radar
-- Diversity Means Superiority

Jian Li
Department of Electrical and Computer Engineering
University of Florida
Gainesville, Florida, USA

Petre Stoica
Department of Information Technology,
Uppsala University,
Uppsala, Sweden

Collaborators: William Roberts, Yao Xie, Luzhou Xu
Outline

- Introduction
- Parameter Identifiability
  - Generic sufficient conditions
  - Cramer-Rao bounds
  - Least-squares estimator
- Adaptive MIMO Techniques
  - Capon
  - Generalized Likelihood Ratio Test
- Flexible Transmit Beampattern Designs
  - Beampattern matching Design
  - Minimum Sidelobe Beampattern Design
  - Comparison with Phased-Array Counterpart
  - Applications
- Summary
Introduction

- **MIMO Radar**
  - Multiple *Different* Transmitted Waveforms
    - Either correlated or uncorrelated

- **Phased-Array Radar**
  - *Single* Waveform Scaled and Transmitted
  - Can be MIMO
  - Special Case:
    - SIMO: Single-Input Multiple-Output
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Generic Sufficient Conditions

- Parameter Identifiability: Maximum number of targets that can be uniquely identified
  - High SNR
  - Large Snapshot Number

- Phased-Array:

\[ K_{\text{max}} < \frac{2M_r}{3} \]

- MIMO Radar:

\[ K_{\text{max}} \in \left( \frac{2(M_t + M_r - 1)}{3}, \frac{2M_tM_r}{3} \right) \]

- Depending on array geometry and the number of shared antennas between transmit and receive arrays.
The maximum number of targets that can be uniquely identified by a MIMO radar is up to $M_t$ times of its phased-array counterpart.

$M_t = \text{Transmit Antenna Number}$
Example 1

\[ M_t = M_r = 10 \]

\[ N = 256 \]

**ULA**

**Half-wavelength spacing**

**Omnidirectional probing (orthogonal waveforms)**

\[ \theta_1 = 0^\circ, \ \theta_2 = 10^\circ, \ \theta_3 = -10^\circ, \ \theta_4 = 20^\circ \ldots \]

\[ \beta_1 = \cdots = \beta_K = 1 \]

**SNR = 20 dB**

A jammer at 45°,

with INR = 20 dB.
Example 1: Cramer Rao Bound

- Phased-array:
  \[ K_{\text{max}} < \frac{2M}{3} = 6.6 \]

- MIMO radar:
  \[ K_{\text{max}} < \frac{2(M_t + M_r - 1)}{3} = 12.7 \]

CRB of first target location vs. target number.
Example 1: LS Estimator

All 12 target locations can be approximately determined from peak locations.
## Example 2

Receiving ULA: 0.5-wavelength spacing
Transmitting ULA: 2.5-wavelength spacing

Orthogonal waveforms

\[
M_t = M_r = 5 \\
\theta_1 = 0^\circ, \ \theta_2 = 8^\circ, \ \theta_3 = -8^\circ, \ \theta_4 = 16^\circ \cdots \\
\beta_1 = \cdots = \beta_K = 1 \\
\text{SNR} = 20 \ \text{dB} \\
\text{A jammer at } 45^\circ, \ \text{INR} = 20 \text{dB}. \\
N = 256
\]
Example 2: Cramer-Rao Bound

Consistent with theoretical analysis:

- Phased-array:
  \[ K_{\text{max}} < \frac{2M}{3} = 3.3 \]

- MIMO radar:
  \[ K_{\text{max}} < \frac{2M_{r}M_{t}}{3} = 16.7 \]

CRB of first target location vs. target number.
Example 2: Least Squares Estimator

All 16 target locations can be approximately determined from peak locations.
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Adaptive MIMO Radar

- **MIMO Radar**
  - Different Targets Reflect \textit{Different} Waveforms
  - Adaptive techniques directly applicable

\[
\tilde{A} = \begin{bmatrix}
\beta_1^* a(\theta_1) & \beta_2^* a(\theta_2) & \cdots & \beta_K^* a(\theta_K)
\end{bmatrix}
\]

\[\tilde{A}^* \tilde{R}_{xx} \tilde{A} = \text{covariance matrix of reflected waveforms}\]

- **Phased-Array**
  - Different Targets Reflect \textit{Identical} Waveform
  - Adaptive Techniques Not Directly Applicable
Example 3

\[ M_t = M_r = 10 \]
\[ N = 256 \]

- Three Targets at -40, 0, 40 degrees
- One Strong Jammer at 25 degrees
- Jammer Waveform Uncorrelated with Radar Transmitted Waveforms
- Uncorrelated Radar Transmitted Waveforms
  - Omnidirectional Probing
- ULA
  - Half-wavelength spacing

\( \times \) : Transmitting antenna,
\( \circ \) : Receiving antenna.
Example 3: Capon and GLRT

Capon Spatial Spectrum

GLRT Pseudo-Spectrum

No false peak
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Transmit Beampattern

- **Covariance Matrix of Transmitted Waveforms**

\[ R = E\{x(n)x^*(n)\} \]

- **Transmit Beampattern**

\[ P(\theta) = a^*(\theta)Ra(\theta) \]

- **Uniform Elemental Power Constraint**

\[ R_{mm} = \frac{c}{M}, \quad m = 1, \cdots, M; \quad \text{with } c \text{ given} \]
Beampattern Matching Design

- **Design Goals**
  - Minimize difference between synthesized and desired beampatterns
  - Minimize cross-correlations among target reflected waveforms
  - Constraint
    - Total power \textit{or}
    - (Equal) elemental power

- Can be formulated as
  Semi-definite Quadratic Program (SQP)

\[
\begin{align*}
\min_{\alpha, \mathcal{R}} & \quad \left\{ \frac{1}{L} \sum_{l=1}^{L} w_l [\alpha \phi(\mu_l) - a^*(\mu_l) \mathcal{R} a(\mu_l)]^2 \\
& \quad + \frac{2w_c}{\bar{K}^2 - \bar{K}} \sum_{k=1}^{\bar{K}-1} \sum_{p=k+1}^{\bar{K}} \left| a^*(\hat{\theta}_k) \mathcal{R} a(\hat{\theta}_p) \right|^2 \right\} \\
\text{s.t.} & \quad R_{mm} = \frac{c}{M}, \quad m = 1, \cdots, M \\
& \quad \mathcal{R} \geq 0,
\end{align*}
\]
Several Examples

- $M=10$
- Equal Elemental Power
- Total Power = 1
- ULA
- Half-wavelength spacing
Example 3: Further Probing

- **Blockdiagram**

  - **Initial probing**
  - **Initial Omnidirectional Probing**
    - GLRT
    - Capon
  - **Angle/Amplitude Estimation**
  - **Beampattern Matching**
    - Enhanced Angle/Amplitude Estimation
      - AML
      - Capon
  - **Optimal Probing**
Example 3: MSE Improvement

Angle MSE vs. Reciprocal of Noise Power

10+ dB

Amplitude MSE vs. Reciprocal of Noise Power

10+ dB

- \( M=10 \), ULA, Half-wavelength spacing
- Equal Elemental Power, Total Power = 1
- Targets at -40, 0, 40 degrees; Jammer at 25 degrees
Example 3: Resolution

Capon

GLRT

- Targets now at -40, 0, 3 degrees

Omnidirectional

Beampattern Matching
Beampattern Matching

**MIMO vs. Phased-Array**

- Phased-Array counterpart no longer convex optimization problem
- MIMO design is Semi-Definite Relaxation (SDR) of Phased-Array design
  - Can be used as initial condition of Newton-type search.

- $M=10$
- Equal Elemental Power
- Total Power = 1
- ULA
- Half-wavelength spacing

**Graphs**

- **MIMO**
  - DOF = 90

- **Phased-array**
  - DOF = 9
Minimum Sidelobe Beampattern Design

Design Goal

- Minimize peak sidelobe level
- Achieve prescribed 3 dB main-beam width
- Subject to either total power or (equal) elemental power constraint

Can be formulated as
Semi-Definite Program (SDP)

MIMO Design is SDR of Phased-Array Design

- MIMO design is used as initial condition for Newton-type search to obtain phased-array design

\[
\min_{t,R} \quad -t \\
\text{s.t.} \quad a^*(\theta_0)Ra(\theta_0) - a^*(\mu_l)Ra(\mu_l) \geq t, \quad \forall \mu_l \in \Omega \\
a^*(\theta_1)Ra(\theta_1) = 0.5a^*(\theta_0)Ra(\theta_0) \\
a^*(\theta_2)Ra(\theta_2) = 0.5a^*(\theta_0)Ra(\theta_0) \\
R \geq 0 \\
R_{mm} = \frac{c}{M}, \quad m = 1, \ldots, M
\]
Minimum Sidelobe Design Example

- \( M=10 \) (uniform elemental power constraint, 0.1 per element)
- Mainbeam pointing at \( \theta_0 = 0^\circ \)
- 3 dB Mainbeam width = 20 degrees
- Sidelobe region \([-90^\circ, -20^\circ] \cup [20^\circ, 90^\circ]\)
- ULA, Half-wavelength spacing
Minimum Sidelobe Design Example

- **Relaxed Elemental Power Constraint**
  - Within 20% of 0.1 for each elemental power
  - Total transmitted power = 1

**Diagram:**

- **MIMO**
- **Phased-array**
Directed Energy System for Homeland Defense

- All antennas transmit at their maximum power
- Large aperture
- Sufficient mainbeam width since target is moving
- No impact on friendly targets
Thermal Therapy System in Medicine

- Ultrasound has good penetration
- All transducers transmit at a low power to avoid harming healthy tissue
- Large aperture needed to deliver sufficient energy
- Sufficient mainbeam width needed
Whale stranded during a military sonar exercise near Canary Islands in July 2004 [1]

“It’s clear that the ocean is under siege from a lot of different sources.”

Currently, 220 dB pulses generated by active sonar and air-gun arrays [1]

- **Distributed Sensor Network?**
  - Large Aperture to Deliver Sufficient Energy At Focal Point
  - Low Power Transmitted by Each Sensor
  - Waveform Diversity to Achieve Desired Mainbeam Width

[1] Source: [Link to Article or Source Material]
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- **Waveform Diversity Offered by MIMO Radar Enables Significant Superiority Over Its Phased-Array Counterpart**
  - Parameter Identifiability
  - Direct Applicability of Adaptive Techniques
  - Flexible Transmit Beampattern Designs

- **MIMO Applications are Diverse**
  - Defense
  - Medicine

- **Probing Further**
  - Many fundamental issues need to be addressed on MIMO
  - Your thoughts?