Super-resolution Technique in TOA Estimation for Precise Geolocation

Dr. Gary F. Hatke, Dr. Patrick R. Marchand

The Tenth ASAP Workshop

12-14 March 2002

This work was sponsored by DARPA under Air Force contract F19628-00-C-0002. Opinions, interpretations, conclusions, and recommendations are those of the authors and not necessarily endorsed by the United States government.
WolfPack Program Vision

- Limitations of Conventional TOA Estimation Techniques
- Description of Super-resolution TOA Algorithm
- Performance with Synthetic and Real Data
• **Concept:**
  Deny enemy radio comm. without disrupting Blue Force

• **Approach:**
  Close-in, distributed, autonomous ground-based system

• **Users:**
  Early entry forces, Special Operations Forces, Army & Marine Corps
WolfPack Program Vision

• Concept:
  Deny enemy radio comm. without disrupting Blue Force

• Approach:
  Close-in, distributed, autonomous ground-based system

• Users:
  Early entry forces, Special Operations Forces, Army & Marine Corps

• Top technical challenge
  Near real-time precision geolocation of tactical RF emitters

• Precision geolocation accuracy:
  Goal 50 m; desired 10 m

• Constraints on node sensor array
  May have imperfect array knowledge, or no array knowledge
Outline

- WolfPack Program Vision

  ⇨ Limitations of Conventional TOA Estimation Techniques

- Description of Super-resolution TOA Algorithm

- Performance with Synthetic and Real Data
Geolocation in Multipath Environment

**Urban Environment**

- Strong multipath scattering; multipath may be stronger than direct path or no direct path
- Large multipath delays (μs)
- Biased TOA estimates cause biased geolocation estimates

**Multipath Channel Model**

[Diagram showing multipath signals and delays]
Geolocation in Multipath Environment

Urban Environment
- Strong multipath scattering; multipath may be stronger than direct path or no direct path
- Large multipath delays (μs)
- Biased TOA estimates cause biased geolocation estimates

Multipath Channel Model

Geolocation based on TDOA
Geolocation in Multipath Environment

**Urban Environment**
- Strong multipath scattering; multipath may be stronger than direct path or no direct path
- Large multipath delays (\(\mu\)s)
- Biased TOA estimates cause biased geolocation estimates

**Multipath Channel Model**

**Correlation-based TOA Estimation**
- Optimal in the absence of multipath:
  \[
  \max_{\tau} \left| R_{XS}(\tau) \right|^2, \text{ where } R_{XS}(\tau) = E \left[ X(t) S^*(t-\tau) \right]
  \]
- Resolution is 1/signal bandwidth. For IS-95 (CDMA) \(B \approx 1.23\) MHz \(\Rightarrow\) biases up to approx. 200m

For precise urban geolocation, conventional correlation-based TOA estimation is insufficient
Illustration of Correlation-based Geolocation
Data from the DARPA Novel Antenna Program

- Large sparse 12 antenna Vivaldi Array, (47.5 λ) located on MIT West Garage roof
- Vertical-blind array at Learning Center
- IS-95 Signals CDMA; B = 1.23 MHz
- Hyperbolas correspond to different antennas at West Garage
Illustration of Correlation-based Geolocation
Data from the DARPA Novel Antenna Program

- Large sparse 12 antenna Vivaldi Array, (47.5 $\lambda$) located on MIT West Garage roof
- Vertical-blind array at Learning Center
- IS-95 Signals CDMA; $B = 1.23$ MHz
- Hyperbolas correspond to different antennas at West Garage

Time in microseconds

Power (dB)

Emitter N1

Emitter N6

350 m

2.08 km

650 m

MIT West Garage

Learning Center

ASAP; SuperResolutionTOA; VG 10
Marchand - 5/10/02
Outline

- WolfPack Program Vision
- Limitations of Conventional TOA Estimation Techniques
  ⇒ **Description of Super Resolution TOA algorithms**
    - Single sensor at WolfPack node (TDOA)
    - Sensor array at WolfPack node (TDOA, AOA)
      • With array manifold knowledge
      • Without array manifold knowledge
- Performance with Synthetic and Real Data
Super-resolution TOA Algorithm
Single Sensor at WolfPack Node

Optimal TOA Statistic w.o. Multipath
\[
\max_{\tau} \left| X^H(\omega) \ D(\tau) \ S(\omega) \right|^2 \quad \text{where}
\]

\[
X(\omega), S(\omega) \text{ are DFTs of } x(t), s(t)
\]

\[
D(\tau) \text{ diagonal matrix, phase ramp of slope } \tau
\]

\[
\max_{\tau} \ d^H(\tau) \left[ (X(\omega) \odot S^*(\omega)) \ (X(\omega) \odot S^*(\omega))^H \right] d(\tau)
\]

where \( d(\tau) \) Vandermonde (diagonal elements of \( D(\tau) \))
Super-resolution TOA Algorithm
Single Sensor at WolfPack Node

Optimal TOA Statistic w.o. Multipath
\[
\max_\tau |X^H(\omega) D(\tau) S(\omega)|^2 \quad \text{where}
\]
\[
X(\omega), S(\omega) \text{ are DFTs of } x(t), s(t)
\]
\[
D(\tau) \text{ diagonal matrix, phase ramp of slope } \tau
\]

Equivalent to
\[
\max_\tau d^H(\tau) [(X(\omega) \circ S^*(\omega)) (X(\omega) \circ S^*(\omega))^H] d(\tau)
\]
where \(d(\tau)\) Vandermonde (diagonal elements of \(D(\tau)\))

Beam-scan Algorithm Spectrum
\[
\begin{align*}
\text{R} & \leftrightarrow (X(\omega) \circ S^*(\omega)) (X(\omega) \circ S^*(\omega))^H \\ 
\text{v} & \leftrightarrow d(\tau)
\end{align*}
\]

Similar to

Computationally efficient super-resolution techniques may be applied when multipath signals “appear” decorrelated in \(R\)
Super-resolution TOA: Block Diagram

Single Sensor at WolfPack Node

Functional Block Diagram

\[ X \xrightarrow{\text{Generate SOI's Temporal Structure Matrix}} S = (S_0^H S_0 S_0^H)^{-1} S \]

\[ S_0 \xrightarrow{\text{Correlation Processor}} C \xrightarrow{\text{DFT Processor}} Q = X(\omega) \odot S^*(\omega) \]

\[ S_0 \xrightarrow{\text{“Shaping” Filter of Temporal Structure Matrix}} \]

\[ \text{Smoothing + Forw./Back. Averaging} \]

\[ \text{Signal/Noise Eigenspace decomposition} \]

\[ \text{Generate Frequency Array Manifold} \]

\[ V(\tau) \xrightarrow{\text{Compute MUSIC-like metrics}} \]

\[ \text{Delay structure } (\tau_1, \tau_2, \ldots, \tau_L) \]

\( \odot \) Hadamard product
Super-resolution TOA
Single Sensor at WolfPack Node

Synthesized IS-95 Signals

\[ \tau_2 = 0.24 \mu s \quad \text{dB} \quad \theta_2 = 30^\circ \]

\[ \tau_1 = 0 \quad \theta_1 = 0^\circ \]

\[ \tau_3 = 2.36 \mu s \quad \theta_3 = 45^\circ \]

Relative Delay (microseconds)

Crosscorrelation (dB)

Truth
Super-resolution TOA
Single Sensor at WolfPack Node

Synthesized IS-95 Signals

Relative Time Delay (microseconds)

Relative Delay (microseconds)

Crosscorrelation (dB)

B = 1.23MHz
Pre-processing SNRs: = (-10, 0, 0) dB

MIT Lincoln Laboratory
Benefits of Sensor Arrays at WolfPack Node

- **Added spatial dimension** enhances multipath resolution
- **Spatial smoothing** improves multipath decorrelation for well-calibrated arrays
- **AOA estimation** at WolfPack node level aids geolocation

**FOCUS:**
- Super resolution algorithm with array manifold known
- Super resolution algorithm without any array manifold knowledge

Geolocation based on TDOA and AOA
Super-resolution TOA: Block Diagram

Single Sensor at WolfPack Node

Functional Block Diagram

\[ X \rightarrow \text{Correlation Processor} \rightarrow C \rightarrow \text{DFT Processor} \rightarrow Q = X(\omega) \odot S^*(\omega) \]

- \[ S = (S_0^H(S_0S_0^H)^{-1})^H \]

- Generate SOI's Temporal Structure Matrix
- Normalize SOI's Temporal Structure Matrix
- Select "Looks" in Frequency domain
- Smoothing + Forw./Back. Averaging
- Signal/Noise Eigenspace decomposition
- Compute MUSIC-like metrics
- Generate Frequency Array Manifold
- V(\tau)

SOI's reference signal

Delay structure \((\tau_1, \tau_2, \ldots, \tau_L)\)
Super-resolution TOA: Block Diagram
Sensor Array at WolfPack Node, Array Manifold Known

Functional Block Diagram

\[ \mathbf{X} \]

Correlation Processor

\[ \mathbf{S} = (\mathbf{S}_0^H \mathbf{S}_0 \mathbf{S}_0^H)^{-1} \mathbf{H} \]

DFT Processor

\[ \mathbf{C} \]

Select “Looks” in Space/Freq. domain

\[ \mathbf{Q} = \mathbf{X}(\omega) \odot \mathbf{S}^*(\omega) \]

Normalize SOI’s Temporal Structure Matrix

\[ \mathbf{S}_0 \]

Generate SOI’s Temporal Structure Matrix

Sub-aperture

Smoothing + Forw./Back. Averaging

Signal/Noise Eigenspace decomposition

Generate Space/Frequency Array Manifold

\[ \mathbf{V}(\theta, \tau) \]

Compute MUSIC-like metrics

SOI’s reference signal

Delay structure \((\tau_1, \tau_2, \ldots, \tau_L)\)

Sensor array knowledge

\[ \mathbf{S}_0 \]
Decorrelation of Multipath

- “Looks” are formed by
  - Considering all overlapping rectangular subsets of $Q$
  - Unwrapping them as vectors

- Covariance matrix is computed from “looks”

- Sub-apertures:
  Frequency: 70%
  Space: 70% for uniform arrays; otherwise 100%
Super-resolution TOA: Block Diagram

Sensor Array at WolfPack Node, Array Manifold Known

Functional Block Diagram

\[ X \xrightarrow{\text{Generate SOI's Temporal Structure Matrix}} S_0 \xrightarrow{\text{SOI's reference signal}} \]

\[ S = (S_0^H(S_0S_0^H)^{-1})^H \]

\[ Q = X(\omega) \odot S^*(\omega) \]

Sensor array knowledge

Delay structure \((\tau_1, \tau_2, \ldots, \tau_L)\)
Super-resolution TOA: Block Diagram

Sensor Array at WolfPack Node, Array Manifold not Known

Functional Block Diagram

SOI’s reference signal

Delay structure \((\tau_1, \tau_2, \ldots, \tau_L)\)

\(\bigotimes\) is Hadamard product

\(\otimes\) is Knonecker product

\(\mathbf{X}\) \(\rightarrow\) Correlation Processor \(\rightarrow\) DFT Processor

\(\mathbf{C}\)

\(\mathbf{S} = (\mathbf{S}_0^H(\mathbf{S}_0 \mathbf{S}_0^H)^{-1})^H\)

Normalize SOI’s Temporal Structure Matrix

Select “Looks” in Space/Freq. domain

Smoothing + Forw./Back. Averaging

Signal/Noise Eigenspace decomposition

Generate Frequency Array Manifold

\(\mathbf{V}(\tau)\)

\(\min \| \Gamma^H \left[ \mathbf{W}(\tau)^H \mathbf{P}_X^{-1} \mathbf{W}(\tau) \right] \Gamma \| \quad \Gamma, \| \gamma_i \| = 1\)

Optimization for \((\tau_1, \ldots, \tau_L)\)

\(\mathbf{X}(\omega) \otimes \mathbf{S}^*(\omega)\)
Outline

- WolfPack Program Vision
- Limitations of Conventional TOA Estimation Techniques
- Description of Super-resolution TOA Algorithm (choose name)

⇒ Performance with Synthetic and Real Data
Eight-Sensor Random “Net” Array
Geolocation with Known Array Manifold

- **SNRs:** (0 dB, 10 dB, 10 dB)
- **Sub-apertures:** Freq. 71% ; Space 100%

Antenna positions are shown as purple stars.
Eight-Sensor Random “Net” Array

Geolocation with Known Array Manifold

- SNRs: (0 dB, 10 dB, 10 dB)
- Sub-apertures: Freq. 71%; Space 100%

Excellent performance with a calibrated array

B = 1.23MHz

Truth: X marks
Random 8-Antenna Array, 10-lambda square

Antenna positions are shown as purple stars
Eight-Sensor Random “Net” Array
Geolocation Sensitivity to Calibration Phase Errors

TOA Estimate

AOA Estimate

TOA performance robust with calibration phase errors
AOA performance drops for calibration phase errors > 20 degrees
Eight-Sensor Random “Net” Array
Geolocation Sensitivity to Sensor Position Errors

Position errors: 0.5 λ (0.175m)

Position errors: 1 λ (0.35 m)

TOA & AOA Performance drops for sensor position errors > .5 λ
Eight-Sensor Random “Net” Array
TOA with Unknown Array Manifold

- Random “Net” Array: 1 sensor
- Sub-apertures: 71% in frequency, 100% in space
- SNR values: 0dB, 10dB, 10dB

![“Net” array image]

Relative Time Delay (microseconds)

TOA Spectrum (dB)

Truth: dashed red lines
Pre-processing SNRs (0dB, 10dB, 10dB)

Number of Sensors Utilized

ASAP; SuperResolutionTOA; VG 28
Marchand - 5/10/02
Eight-Sensor Random “Net” Array
TOA with Unknown Array Manifold

- Random “Net” Array: 1 to 8 sensors
- Sub-apertures:
  - 71% in frequency
  - 100% in space
- SNR values:
  - 0dB, 10 dB, 10 dB

“Net” array

![Diagram of 'Net' array with sensors and sub-apertures]
Super-resolution Array-based TOA
NAP Data; Emitter N5

Learning Center:
2-antenna elements sub-array #3

Vertical-Blind Array
Sub-array #3

Large Sparse Vivaldi Array
Sub-array #1
Sub-array #2

MIT West Garage:
2-antenna elements sub-array #1
2-antenna elements sub-array #2
Super-resolution Array-based TOA
NAP Data; Emitter N5

Differential path delay = 0.21 μs

MIT West Garage: 2-antenna elements sub-array #1

2-antenna elements sub-array #2

Optical fiber delay = 3.68 μs

Learning Center: 2-antenna elements sub-array #3

1554 m

1618 m
Super-resolution Array-based TOA
NAP Data; Emitter N5; Array Manifold not Known

Conventional correlation-based processing

![Graph showing correlation-based processing]

<table>
<thead>
<tr>
<th>Correlation-based errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4.07 - 0.21 - 3.68 = 0.18 \mu s$</td>
</tr>
</tbody>
</table>
Super-resolution Array-based TOA

NAP Data; Emitter N5; Array Manifold not Known

---

**Correlation-based errors**

\[ 4.07 - 0.21 - 3.68 = 0.18 \mu s \]

**Approx. error reduction**

Factor of 4

**Super-resolution errors**

\[ 3.85 - 0.21 - 3.68 = -0.04 \mu s \]
Super-resolution Array-based TOA
NAP Data; Emitter N5; Array Manifold not Known

<table>
<thead>
<tr>
<th>Correlation-based errors</th>
<th>Approx. error reduction</th>
<th>Super-resolution errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.07 – 0.21 – 3.68 = 0.18 μs</td>
<td>Factor of 4</td>
<td>3.85 – 0.21 – 3.68 = – 0.04 μs</td>
</tr>
<tr>
<td>3.06 – 0.21 – 3.68 = – 0.83 μs</td>
<td>Factor of 3</td>
<td>3.58 – 0.21 – 3.68 = – 0.31 μs</td>
</tr>
</tbody>
</table>
Super-resolution Array-based TOA

NAP Data; Emitter N5; Array Manifold not Known

**Correlation-based errors**

<table>
<thead>
<tr>
<th>Approx. error reduction</th>
<th>Super-resolution errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.07 – 0.21 – 3.68 = 0.18 µs</td>
<td>3.85 – 0.21 – 3.68 = −0.04 µs</td>
</tr>
<tr>
<td>3.06 – 0.21 – 3.68 = −0.83 µs</td>
<td>3.58 – 0.21 – 3.68 = −0.31 µs</td>
</tr>
<tr>
<td>rms error = 0.6 µs (180 m)</td>
<td>rms error = 0.22 µs (66 m)</td>
</tr>
</tbody>
</table>

**Approx. error reduction**

| Factor of 4 | Factor of 3 | Factor of 3 |

**Relative Power (dB)**

Conventional correlation-based processing

Super-resolution array-based processing

**Notes:**

- rms error = 0.6 µs (180 m)
- rms error = 0.22 µs (66 m)
Conclusion

- New super-resolution array-based technique for emitter geolocation
  - Applicable to single sensors or sensor arrays
  - Takes advantage of sensor array knowledge if available but does not require it

- Performance with imperfect array knowledge
  - TOA performance is robust with phase calibration errors
  - AOA performance useful if phase calibration errors < 30°
  - TOA and AOA performances maintained if sensor position errors < λ/2
Conclusion

- **New super-resolution array-based technique for emitter geolocation**
  - Applicable to single sensors or sensor arrays
  - Takes advantage of sensor array knowledge if available but does not require it

- **Performance with imperfect array knowledge**
  - TOA performance is robust with phase calibration errors
  - AOA performance useful if phase calibration errors < 30°
  - TOA and AOA performances maintained if sensor position errors < \( \lambda/2 \)

- **Performance without any array knowledge**
  - Compared to correlation-based techniques, the new super-resolution technique showed roughly a threefold improvement in geolocation accuracy with NAP data
Conclusion

- **New super-resolution array-based technique for emitter geolocation**
  - Applicable to single sensors or sensor arrays
  - Takes advantage of sensor array knowledge if available but does not require it

- **Performance with imperfect array knowledge**
  - TOA performance is robust with phase calibration errors
  - AOA performance useful if phase calibration errors $< 30^\circ$
  - TOA and AOA performances maintained if sensor position errors $< \lambda/2$

- **Performance without any array knowledge**
  - Compared to correlation-based techniques, the new super-resolution technique showed roughly a threefold improvement in geolocation accuracy with NAP data

- **Need to characterize performance more fully with “WolfPack data”**