Space-Time Adaptive Processing Using Sparse Arrays

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Application: Space Based Radar

- Fast orbital velocity (Large aperture ~ GMTI performance)
- Long range to target (Large aperture ~ location accuracy)
- Launch cost
  ~low weight and size (folded)

DARPA Erectorsat Program:
Assembly of large radar apertures in space
Outline

• Introduction
• Theory
• Performance
• Summary
STAP Units

\[ D = \frac{2v \sin(\theta)}{\lambda} \]
STAP Units

Velocity

Normalized Doppler (Rel. PRF)

SIN (Azimuth)
Doppler Ambiguous Clutter

\[ \beta = \frac{4v}{\lambda PRF} \]

Main Beam Clutter Width \( = \frac{\lambda 2v}{L} \text{ (m/s)} = \frac{4v}{L} \text{ (Hz)} \)

Beamwidth

Fast Platform
Slow Platform
Aperture and Doppler Limited Performance

- Aperture limited performance is reached if the array travels more than one aperture length in a CPI.
- Fast moving platforms (e.g., SBR) need long apertures to achieve resolution limited performance for typical CPI lengths.
- Large arrays are expensive.

[Graphs and diagrams showing SINR loss for different normalized Doppler values and element counts for β=1 and β=4.]
Some Sparse Array Concepts

- Sparse arrays trade mainlobe width against grating lobe height to find the optimum sparseness.
- Energy transferred from the mainlobe to the grating lobes is useless for Tx.
  - Use a filled section of the sparse array for Tx. And form multiple Rx. beams.
Sparse Array Issues

• **Adaptive beamformer / STAP performance**
  – Narrower null due to increased aperture
  – Losses due to grating lobes / nulls

• **Angle estimation performance**
  – Improved accuracy due to narrower beamwidth (CRB)
  – Non-local errors due to grating lobes (WWB, ZZB, AB, …)

• **SAR performance**
  – Multiple spatial samples per pulse
  – Tight PRF constraints

• **Hardware and cost**
  – Sparse arrays require less hardware
  – Cheaper & lighter

• …
Outline

• Introduction

• Theory
  – Clutter Rank
  – Waveforms
  – SINR Loss

• Performance

• Summary
Brennan’s Rule & Ward’s Rules*

- Brennan’s rule for filled arrays:
  \[ r = N + \beta(M - 1) \]
- Ward’s rules for sparse arrays:
  \[ r_{\text{min}} = N + M - 1 \]
  \[ r_{\text{max}} = N_{\text{fill}} + \beta(M - 1) \]

*J. Ward, Asilomar 1998

\[ N = \text{Number of elements}, \quad M = \text{Number of pulses}, \quad \beta = 2 v T d_0^{-1}, \quad N_{\text{fill}} = \text{Number of elements in filled array} \]
Additional Sparse Array Behavior

$N = 24, M = 10, \beta = 4$ Example

- Length = 24 ele.
  - 2 Subarrays
  - 3 Subarrays
  - 4 Subarrays

- Length = 50 ele.
  - 2 Subarrays
  - 3 Subarrays
  - 4 Subarrays

- Length = 80 ele.
  - 2 Subarrays
  - 3 Subarrays
  - 4 Subarrays

Eigenvlaue Index

Eigenvalue (dB)

Clutter Rank

Aperture Length (Element Positions)
New (?) Rules for Sparse Arrays

For arrays which move less than the smallest subarray aperture during a pulse the rank is given by:

\[
\min\left[ N + \beta(M-1) + G , N + S\beta(M-1) \right]
\]

Jim Ward’s \( r_{\text{max}} \)  
Using each sub array independently

For equal size subarrays a sparse array is no better than a single subarray if

\[ G > \beta(S-1)(M-1) \]

I.e., The array is so sparse that there is no redundancy

\( G = \) Sum gap sizes (element positions) \( S = \) Number of subarrays
Sparse Aperture Waveforms

- Ambiguous waveforms (e.g., pulse-Doppler) and sparse (ambiguous) apertures lead to multiple clutter nulls
- Unambiguous waveforms preferable

Clutter Ridge
\[ D = 2 v \sin(\theta) \lambda^{-1} \]
Long Single Pulse Waveforms

- Single pulse means no range or Doppler ambiguities
  - High chip rate sets Doppler ambiguities

- Must pulse compress each Doppler bin separately
  - More computation than pulse-Doppler waveforms

- Concern about strong sidelobe clutter > noise floor
  - Wide bandwidth & narrow antenna beampatterns

Pulse length: up to 20ms @ 3000 km

20 ms = 50 Hz Doppler Resolution = 0.75 m/s Velocity Resolution @ 10 GHz
Processing Long Single Pulse Waveform

- Long single pulse radar can be made to ‘appear’ like a regular pulse-Doppler radar

- Looks like high PRF radar without the range ambiguities
**Space Time Adaptive Processing**

- Grating lobes lead to reduced detection performance at particular Doppler frequencies

\[
\text{SINR Loss} \approx \| \mathbf{v} \|^2 - \left( \| \mathbf{v} \|^2 - \frac{\text{Grating Lobe Gain}}{\text{Mainbeam Gain}} \right) = 1 - \frac{\text{Grating Lobe Gain}}{\text{Mainbeam Gain}}
\]

- Should not make the array too sparse
  - For <3 dB SINR loss grating lobe gain must be 3 dB less than main lobe gain (Σ grating lobes for pulse-Doppler waveforms?)
Outline

• Introduction

• Theory

• Performance
  – Dependence on waveform
  – SBR Design Example

• Summary
Unambiguous vs. Ambiguous Waveforms
Interferometer Example

N = 8, M = 32, $\beta = 1$

- Filled rank = $8 + 1(32-1)$
  = 39
- Max. sparse rank = $8 + 2(32-1)$ = 70 (reached with a 31 element gap)

N = 8, M = 8, $\beta = 4$

- Filled rank = $8 + 4(8-1)$
  = 36
- Runs out of DOFs with a 27 element gap
- $8 + 27 + 2(32-1)$ = 63

Doppler unambiguous waveforms better preserve the available DOFs
Unambiguous vs. Ambiguous Waveforms

- Combination of Doppler and angle ambiguities leads to poor SINR performance

Grating lobes on Doppler ambiguous clutter

Multiple grating lobes on Doppler ambiguous clutter
Space Based GMTI Radar Examples

Parameters

- 32m x 2.5m filled aperture
- 10 GHz operating frequency
- 1000 km orbit
  - 7282 m/s orbital velocity
- 1 kw peak transmit power
- 200 MHz bandwidth
- Unambiguous waveform
- -12 dB const. $\gamma$ clutter model
- 2500 km range
  - 16.67ms CPI length
  - Travel ~120m in a CPI

Scenarios

Area of interest

0° Rotation

60° Rotation

Doppler

SIN (Angle)

MIT Lincoln Laboratory
Space Based Radar GMTI Designs

Interferometer Array

Even Spaced Equal Size

Uneven Spaced Equal Size

Many Apertures

- Many possible array configurations
  - Radar performance
  - Ease of launch and assembly*
  - Mechanical issues* ...

* Issues being addressed by Aerospace Corporation
Many unequal apertures provides the longest array and best performance.
60° Rotation Scenario

Interferometer Array

Three Equal Arrays - Even

Three Equal Arrays - Uneven

Many Unequal Apertures

Better overall MDV, but reduced total baseline in some cases
-3 dB MDV vs. Array Length

0° Rotation

Lower variance of the subarray positions of the many unequal config.

60° Rotation

Interferometer
3 Equal – Even
3 Equal – Uneven
Many Unequal

- Many unequal subarrays configuration needs a larger baseline to obtain the same performance as the other configurations, but ultimately provides the best MDV
  - 165m aperture optimizes MDV for 2500 km range
  - Longer apertures improve angle metrics
Summary

• Sparse arrays potentially improve the minimum detectable performance of space-based radars
  – Approach the MDV performance of a large filled aperture much with lower size, weight and cost

• Sparse arrays and sparse (pulse-Doppler) waveforms do not mix well
  – Sparse arrays perform well with Doppler unambiguous waveforms
  – Sparse waveforms (pulse-Doppler) perform well with filled arrays

• Long single-pulse waveforms provide range and Doppler unambiguous operation and are compatible with current STAP algorithms

• Sparse arrays with many unevenly sized unevenly spaced subarrays provide the best GMTI performance
Interferometer Array Grating Lobes

Fill Fraction = \frac{\text{Filled Aperture}}{\text{Total Aperture}}

- Grating lobes quickly appear for interferometer array
- \sim \frac{2}{3} fill fraction -3 dB grating lobes untapered apertures
Lower grating lobes than interferometer

Higher gap ratios lead to lower grating lobes

Also poorer MDV performance
Grating Lobe Distributions
Unequal Arrays

- Multiple unequal arrays have the best grating lobe performance
0° Rotation Scenario

Interferometer Array

Three Equal Arrays - Even

Three Equal Arrays - Uneven

Many Unequal Apertures

Array Length (m)

Normalized Doppler

Normalized SINR (dB)
Three Equal Apertures Target Location

- 96 m aperture largest possible without increasing the threshold SNR
  - Provides 89 m rms error at 6° grazing
  - 82 m gives 107 m rms error
Three Unequal Apertures Target Location

- 72 m aperture largest possible without increasing the threshold SNR
- 72 m aperture Provides 119m rms error at 6° grazing

Weiss Weinstein Bound

- Gap Ratio: 1:1, 1:2, 1:3, 1:4, 1:5

Graph showing Peak Grating Lobe and Weiss Weinstein Bound.
SINR Loss Due To Grating Lobe
(Spatial Only Example)

20 Element Array Example

• Under the high INR assumption:

\[ \text{SINR Loss} \approx v^H v - |v^H e|^2 = 1 - \frac{\text{Grating Lobe Gain}}{\text{Mainbeam Gain}} \]

• i.e., for 3 dB loss grating lobe gain (sum grating lobes for pulse-Doppler ?) must be 3 dB less than main lobe gain