Wide-Angle SAR Image Formation with Migratory Scattering Centers and Regularization in Hough Space

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Outline

• Spotlight-Mode SAR Imaging
• Joint Image Formation and Anisotropy Characterization
• Overcomplete Basis Formulation
• Sparsifying Regularization
• Moving From Pixels to Objects
  ▪ Migratory Scattering Centers
  ▪ Regularization in Hough Space
2-D Spotlight-Mode SAR

\[
s(x, y)
\]
Observation Model

- Phase history domain:
  \[ r(f, \theta) = \int \int s(x, y) e^{-j \frac{4\pi f}{c} (x \cos \theta + y \sin \theta)} \, dx \, dy \]

- Range profile domain:
  \[ R(\rho, \theta) = \int \int s(x, y) \delta(\rho - x \cos \theta - y \sin \theta) \, dx \, dy \]

  - Projection of scattering function in direction specified by \( \theta \)

- Related by 1-D Fourier transform
  \[ r(f, \theta) = \int_{-L}^{L} \hat{R}(\rho, \theta) \exp \left\{ -j \frac{4\pi f}{c} \rho \right\} d\rho \]
Point Scatterers

• At typical SAR frequencies, scattering appears as though coming from a set of discrete points

\[ s(x, y) = \delta(x_0, y_0) \]

• Phase history:  \[ r(f, \theta) = \exp\left\{ -j \frac{4\pi f}{c} (x_0 \cos \theta + y_0 \sin \theta) \right\} \]

• Range profile:  \[ \hat{R}(\rho, \theta) = \delta(\rho - x_0 \cos \theta - y_0 \sin \theta) \]

• Sinusoid in range profile:  \[ \rho = x_0 \cos \theta + y_0 \sin \theta \]
Wide-Angle Imaging and Anisotropy

- In principle, wide-angle apertures (long flight path) allow formation of images with high cross-range resolution
- Isotropic scattering assumption violated
- Scattering function depends on aspect angle $\theta$

$$s(x, y, \theta)$$
Point-Scatterer Model with Anisotropy

- With $P$ point-scattering centers and with anisotropy, phase history measurements modeled as:

$$r(f, \theta) = \sum_{p=1}^{P} s(x_p, y_p, \theta) \exp \left\{ - j \frac{4\pi f}{c} (x_p \cos \theta + y_p \sin \theta) \right\}$$

- Joint image formation and anisotropy characterization: Recover $s(x,y,\theta)$ from $r(f,\theta)$
Approach

- Spatial locations \((x_p, y_p)\) can be pixels or points of interest
- Expand \(s(x_p, y_p, \theta)\) into an overcomplete basis
- Ill-posed inverse problem
- Solve for coefficients with sparsifying regularization
Overcomplete Basis Expansion

- Expand $s(x_p, y_p, \theta)$ into $\{b_1(\theta), b_2(\theta), \ldots, b_M(\theta)\}$

$$r(f, \theta) = \sum_{p=1}^{P} \sum_{m=1}^{M} a_{p,m} b_m(\theta) \exp\left\{-j \frac{4\pi f}{c} (x_p \cos \theta + y_p \sin \theta)\right\}$$

- Problem: determine unknown coefficients $a_{p,m}$

- $f, \theta$ discrete variables
  - $f_1, f_2, \ldots, f_K$
  - $\theta_1, \theta_2, \ldots, \theta_N$

- Overcomplete basis
  - $M > N$
Matrix-Vector Form

• Expressible in matrix-vector form:

\[
\begin{bmatrix}
   r(f_1, \theta) \\
   r(f_2, \theta) \\
   \vdots \\
   r(f_K, \theta)
\end{bmatrix} =
\begin{bmatrix}
   b_1(\theta)e^{-4 j/\pi f_1(x_1 \cos \theta + y_1 \sin \theta)/c} \\
   b_1(\theta)e^{-4 j/\pi f_2(x_1 \cos \theta + y_1 \sin \theta)/c} \\
   \vdots \\
   b_1(\theta)e^{-4 j/\pi f_K(x_1 \cos \theta + y_1 \sin \theta)/c}
\end{bmatrix} \cdots
\begin{bmatrix}
   b_M(\theta)e^{-4 j/\pi f_1(x_2 \cos \theta + y_2 \sin \theta)/c} \\
   b_M(\theta)e^{-4 j/\pi f_2(x_2 \cos \theta + y_2 \sin \theta)/c} \\
   \vdots \\
   b_M(\theta)e^{-4 j/\pi f_K(x_2 \cos \theta + y_2 \sin \theta)/c}
\end{bmatrix} \begin{bmatrix}
   a_{1,1} \\
   a_{1,M} \\
   \vdots \\
   a_{P,1} \\
   a_{P,M}
\end{bmatrix}
\]

• \( r = \Phi a \)

• \( (r = \Phi a + n) \)
Specific Choice of Basis

- What are the $b_m(\theta)$?
- Choose to allow parsimonious representation
- Contiguous intervals of anisotropy – all widths and all shifts
Specific Choice of Basis

• Rectangular pulses, for example
  ▪ Can seamlessly insert any other shape
  ▪ Hamming, Triangle, Raised Triangle

• Incorporates some prior information
Sparsifying Regularization

- Find \( a \) that minimizes \( J(a) = \| r - \Phi a \|_2^2 + \alpha \| a \|_k^k, k < 1 \)
  - \( \| \cdot \|_k \) denotes \( \ell_k \)-norm
- First term: data fidelity
- Second term: regularization term favoring sparsity
- \( J(a) \) can be minimized effectively
  - Quasi-Newton method [Çetin & Karl 2001]
  - Greedy graph-structured algorithm [Varshney et al. 2006]
Quick Example

<table>
<thead>
<tr>
<th></th>
<th>least-squares solution ($\alpha = 0$)</th>
<th>sparse solution ($\alpha = 3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coefficients</td>
<td>![Graph 1]</td>
<td>![Graph 2]</td>
</tr>
<tr>
<td>$s(\theta)$</td>
<td>![Graph 3]</td>
<td>![Graph 4]</td>
</tr>
</tbody>
</table>
Moving from Pixels to Objects

• Scattering centers have more meaning than just pixels
• More information is available to allow scene interpretation
• 2 phenomena
  - Anisotropy width and spatial extent are related
  - Certain scattering mechanisms migrate as a function of $\theta$
Glint Anisotropy

• Glint or flash – like a mirror
• Very thin response in $\theta$
• Corresponds to long flat plate in space
• Thinner anisotropy, longer spatial extent
• Would like to explain glint anisotropy parsimoniously
  • Additional sparsity along a line
  • Can infer properties about objects in scene such as orientation and spatial extent
Range profile

- $\rho$ parameterizes direction of $\theta$
  in ground plane geometry
- For fixed $\theta$, $\rho = x\cos\theta + y\sin\theta$
Hough Transform Ideas

- Points on a line in image space
- Intersecting sinusoids in range profile space
Sparsity Along A Line

- For sparsity along a line, use Hough transform ideas
- Sparsity among scatterers in $\rho$-$\theta$ cells

\[ f_{Li,j} = S_{i,j}F\Phi' \]

- $\Phi'$ takes coefficients to phase history domain (not exactly $\Phi$)
- $F$, DFT-like operator, converts to range profile domain
- $S_{i,j}$ selects the $(i,j)^{th}$ cell in the range profile domain

\[
J(a) = \|r - \Phi a\|_2^2 + \alpha_0 \|a\|_k + \alpha_1 \sum_{i=1}^{K} \sum_{j=1}^{N} \|L_{i,j}a\|_k^k
\]
Scene with Glint Anisotropy

Conventional image
(formed with polar format algorithm)

True anisotropy at a point
Example: $\alpha_0 = 30$, $\alpha_1 = 0$

parsimonious in basis vectors per pixel
Example: $\alpha_0 = 0, \alpha_1 = 20$

parsimonious in pixels
Example: $\alpha_0 = 30, \alpha_1 = 20$

solution represents entire glint with one basis vector
Other Hough-Type Regularization Terms

• Sparsity along a line is not the only possibility
• Can favor other types of spatial geometry in image formation
Migratory Scattering Centers

- Other effect besides anisotropy prominent in wide-angle SAR
- Certain types of scattering centers migrate as function of $\theta$
- Parameterize migration around a ‘center’
- Scattering center closer to radar by $R(\theta)$
- If circle, $R(\theta) = R$
Trigonometry

- \( s(x, y) = \delta(x_c - R\cos\theta, y_c - R\sin\theta) \)

- \( \rho = (x_c - R\cos\theta)\cos\theta + (y_c - R\sin\theta)\sin\theta \)
  
  \[ = x_c\cos\theta + y_c\sin\theta - R \]

- \( \rho = x_c\cos\theta + y_c\sin\theta - R(\theta) \)
Migratory Scattering Center Observation Model

\[
\begin{align*}
    r(f, \theta) &= \sum_{p=1}^{P} s(\bar{x}_p, \bar{y}_p, \theta) \exp \left\{ -j \frac{4\pi f}{c} \left[ (\bar{x}_p + R_p(0))\cos \theta + \bar{y}_p \sin \theta - R_p(\theta) \right] \right\} \\
    (\bar{x}_p, \bar{y}_p) & \text{ invariant spatial location (when } \theta = 0) \\
\end{align*}
\]
Procedures for Circular Migration Characterization

• Restricting to migration in a circle, two methods

• Method 1: even more overcomplete basis

\[ \sum_{p=1}^{P} \sum_{l=1}^{L} \sum_{m=1}^{M} a_{p,l,m} b_m(\theta) \exp \left\{ -j \frac{4\pi f}{c} \left( (\bar{x}_p + R_{p,l}) \cos \theta + \bar{y}_p \sin \theta - R_{p,l} \right) \right\} \]

• Techniques to minimize cost function unchanged
Procedures for Circular Migration Characterization

• Method 2: optimize over vector of radii $\mathbf{R}$ (one entry per scatterer)

\[ \min_{\mathbf{R}} \| \mathbf{r} - \Phi(\mathbf{R}) \|_2 \arg \min_{\mathbf{a}} \| \mathbf{r} - \Phi(\mathbf{R}) \|_2^2 + \alpha \| \mathbf{a} \|_k^2 \]

• basis vectors as function of radius:

\[ \phi_{k,p,m}(R_p) = b_m(\theta) \cdot \exp \left\{ - j \frac{4 \pi f_k}{c} \left( (\bar{x}_p + R_p) \cos \theta + \bar{y}_p \sin \theta - R_p \right) \right\} \]

• minimization by nonlinear least-squares optimization techniques, where the nonlinear function involves solving the inner sparsifying regularization minimization
Example

true migratory scattering shape overlaid on conventional image

migratory scattering solution overlaid on conventional image

solution gives much more object-level information than conventional image
Non-Circular Migration

- Characterize circular migration over subapertures
Conclusion

• Novel overcomplete basis and sparse signal representation formulation for anisotropy characterization in wide-angle SAR

• Exploit available information in phase history measurements to extract object-level information
  ▪ Hough space regularization for glint anisotropy
  ▪ Migratory basis vectors in overcomplete basis for migratory scattering
Questions