Introduction to Radar Systems

The Radar Equation

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ODonnell 06-13-02

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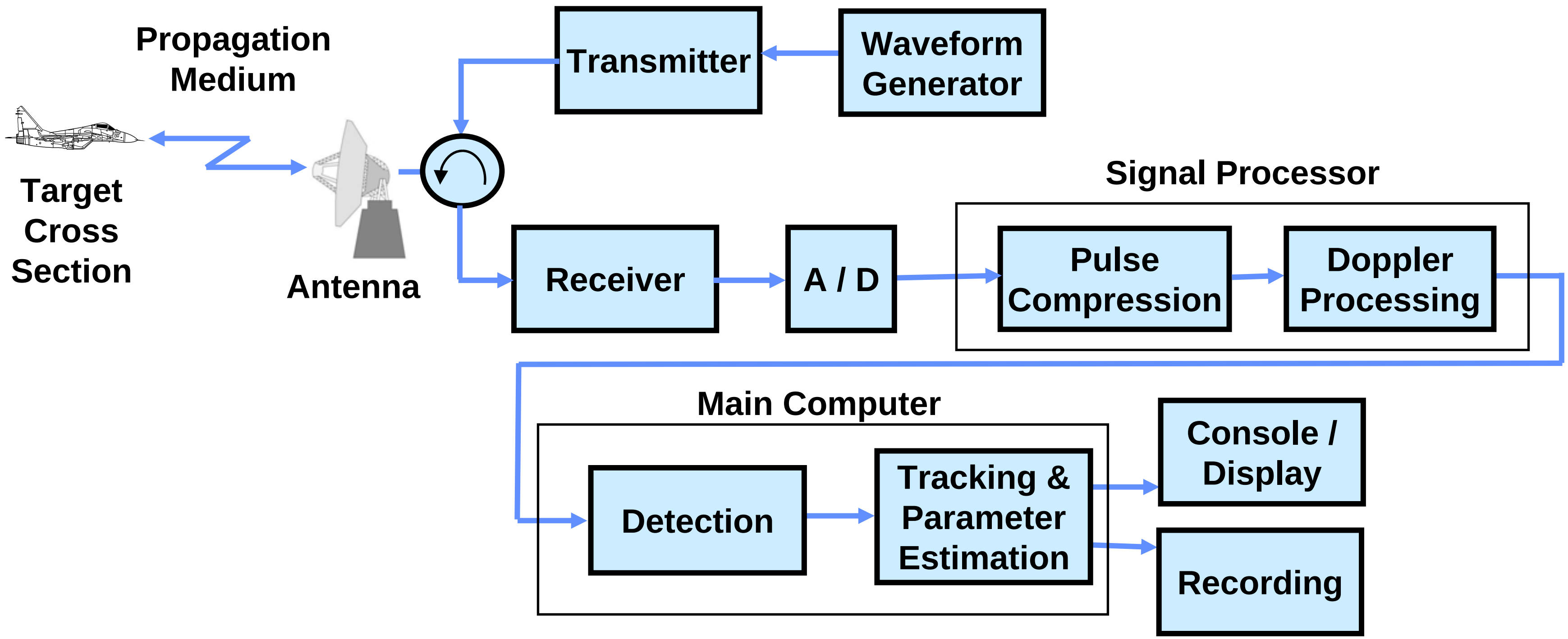
Introduction – The Radar Range Equation



The Radar Range Equation Connects:

1. **Target Properties - e.g. Target Reflectivity (radar cross section)**
2. **Radar Characteristics - e.g. Transmitter Power, Antenna Aperture**
3. **Distance between Target and Radar - e.g. Range**
4. **Properties of the Medium - e.g. Atmospheric Attenuation.**

Outline

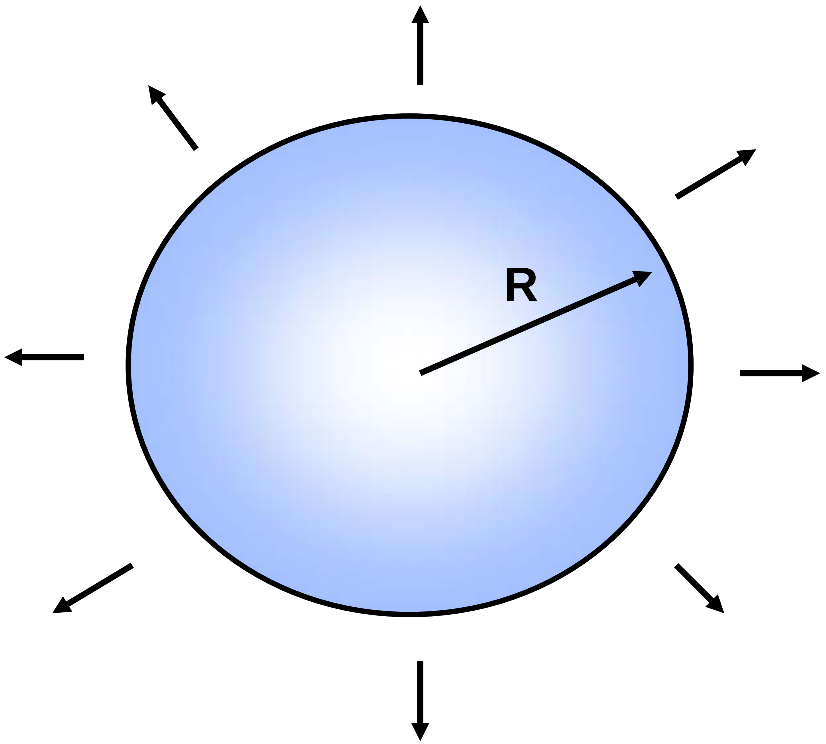
- Introduction
- Introduction to Radar Equation
- Surveillance Form of Radar Equation
- Radar Losses
- Example
- Summary

Radar Range Equation

Power density from uniformly radiating antenna transmitting spherical wave

$$\frac{P_t}{4 \pi R^2}$$

P_t = peak transmitter power
R = distance from radar



Radar Range Equation (continued)

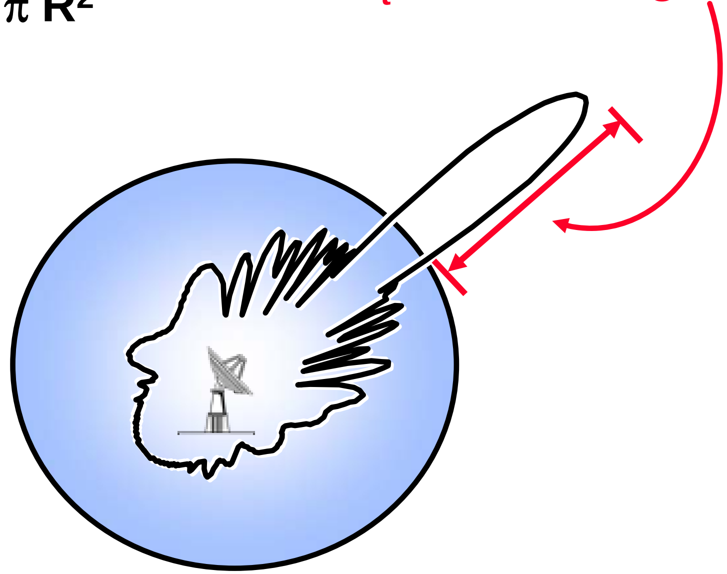
Power density from isotropic antenna $\frac{P_t}{4 \pi R^2}$

Power density from directive antenna $\frac{P_t G_t}{4 \pi R^2}$

P_t = peak transmitter power

R = distance from radar

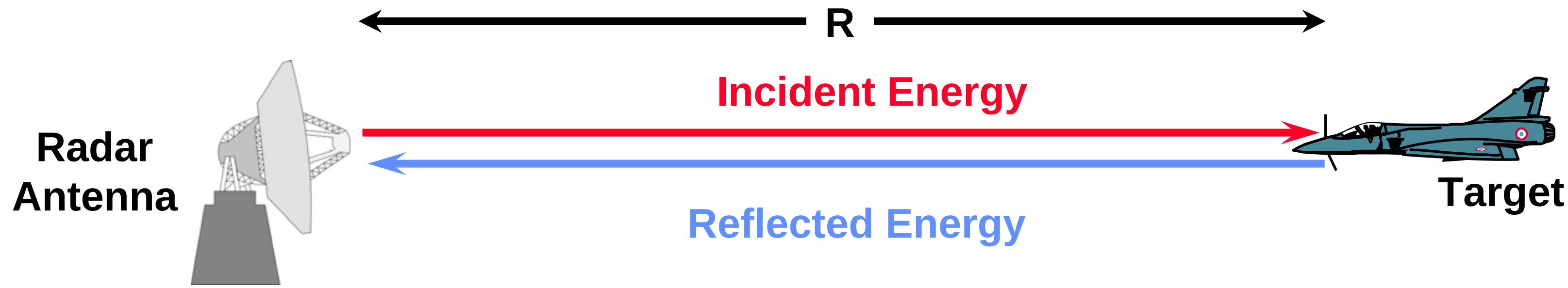
G_t = transmit gain



Gain is the radiation intensity of the antenna in a given direction over that of an isotropic (uniformly radiating) source

$$\text{Gain} = 4 \pi A / \lambda^2$$

Definition of Radar Cross Section (RCS or σ)



Radar Cross Section (RCS or σ) is a measure of the energy that a radar target intercepts and scatters back toward the radar

Power of reflected signal at target

$$\frac{P_t G_t}{4 \pi R^2} \sigma$$

Power density of reflected signal at the radar

$$\frac{P_t G_t}{4 \pi R^2} \frac{\sigma}{4 \pi R^2}$$

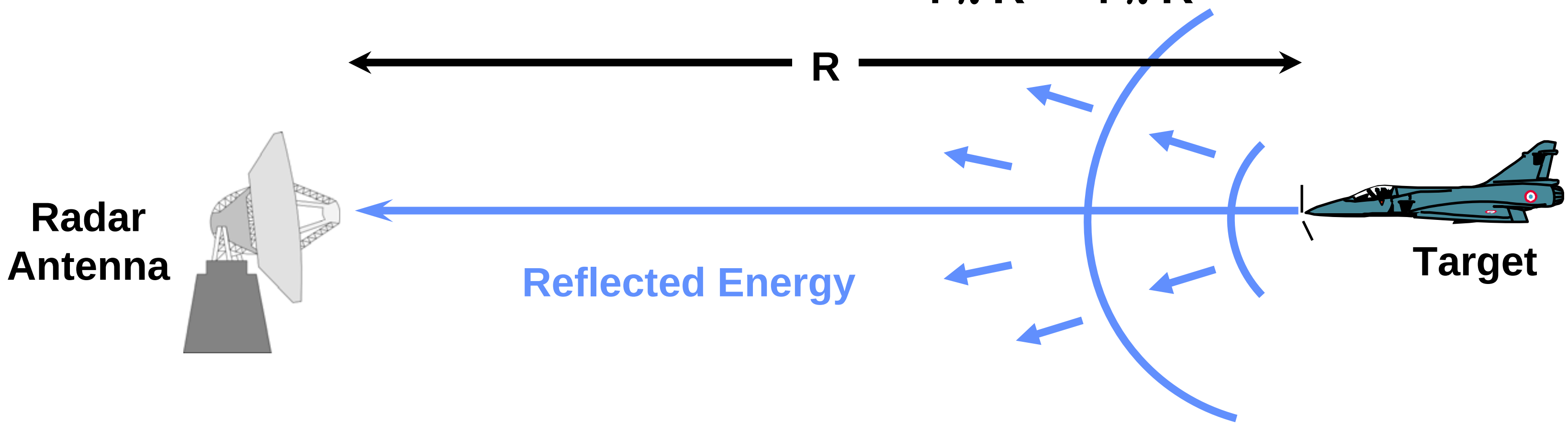
σ = radar cross section units (meters)^2

Power density of reflected signal falls off as $(1/R^2)$

Radar Range Equation (continued)

Power density of reflected signal at radar

$$\frac{P_t G_t}{4 \pi R^2} \frac{\sigma}{4 \pi R^2}$$



The received power = the power density at the radar times the area of the receiving antenna

Power of reflected signal from target and received by radar

$$P_r = \frac{P_t G_t}{4 \pi R^2} \frac{\sigma A_e}{4 \pi R^2}$$

P_r = power received

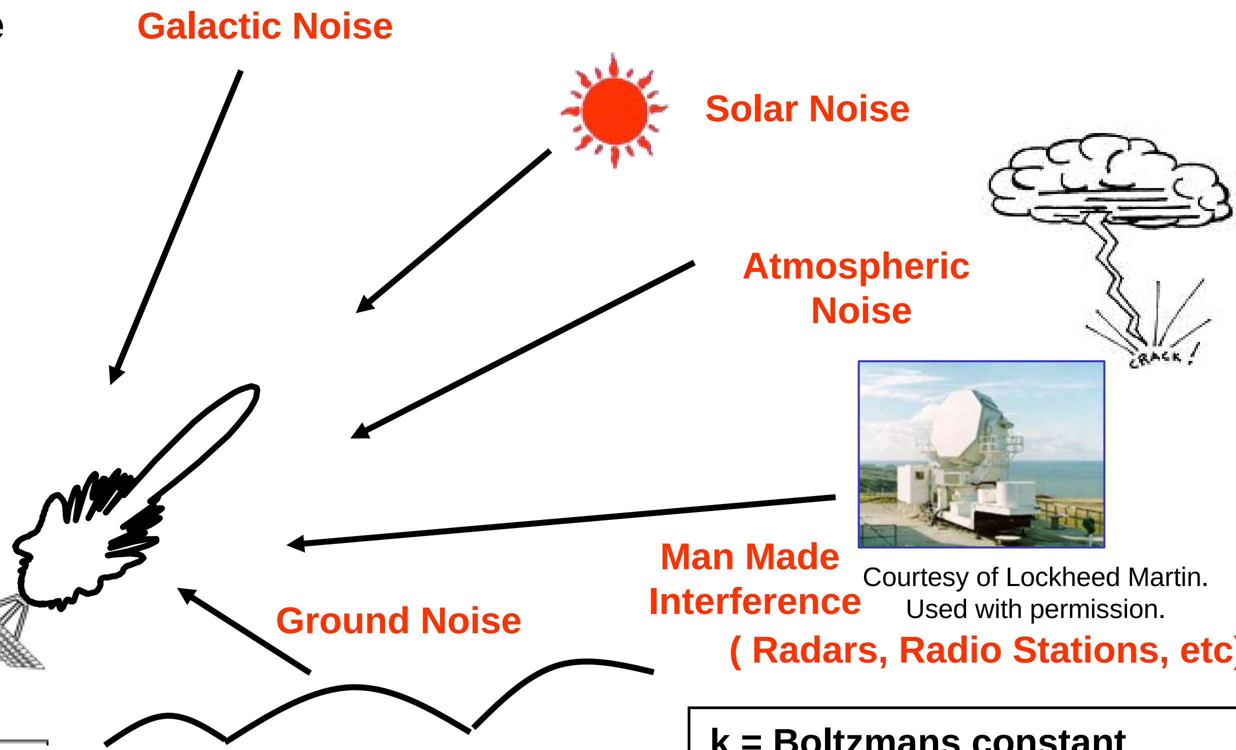
A_e = effective area of receiving antenna

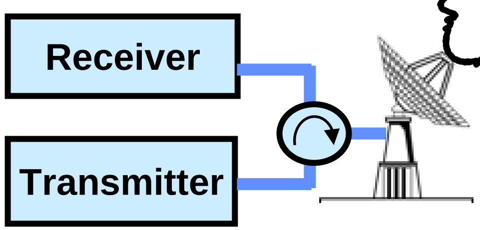
Sources of Noise Received by Radar

- The total effect of these noise sources is represented by a single noise source at the antenna output terminal.

- The noise power at the receiver is given by:

$N = k B_n T_s$





(Receiver, waveguide, and duplexer noise)

Noise from Many Sources Competes with the Target Echo

k = Boltzmans constant
= 1.38 x 10^{-23} joules / deg ^{o}K
T_s= System Noise Temperature
B_n = Noise bandwidth of receiver

Radar Range Equation (continued)

Signal Power reflected from target and received by radar

$$P_r = \frac{P_t G_t}{4 \pi R^2} \frac{\sigma A_e}{4 \pi R^2}$$

Average Noise Power

$$N = k T_s B_n$$

Signal to Noise Ratio

$$S / N = P_r / N$$

Assumptions :
$G_t = G_r$
L = Total System Losses
$T_o = 290^o$ K

$$S / N = \frac{P_t G^2 \lambda^2 \sigma}{(4 \pi)^3 R^4 k T_s B_n L}$$

Signal to Noise Ratio (S/N or SNR) is the standard measure of a radar's ability to detect a given target at a given range from the radar

" S/N = 13 dB on a 1 m^2 target at a range of 1000 km"

radar cross section of target

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System Noise Temperature

- The System Noise Temperature, T_S , is divided into 3 components :

$$T_s = T_a + T_r + L_r\, T_e$$

- T_a is the contribution from the antenna
  - Apparent temperature of sky (from graph)
  - Loss within antenna
- T_r is the contribution from the RF components between the antenna and the receiver
  - Temperature of RF components
- L_r is the loss of input RF components
- T_e is the temperature of the receiver
  - Noise factor of receiver

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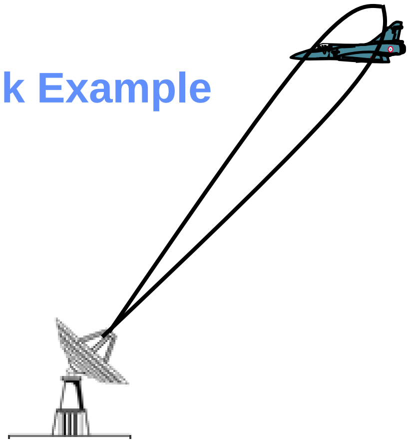
Track Radar Range Equation

Track Radar Equation

$$S / N = \frac{P_t \, G^2 \, \lambda^2 \, \sigma}{(4 \, \pi)^3 \, R^4 \, k \, T_s \, B_n \, L}$$

- When the location of a target is known and the antenna is pointed toward the target.

Track Example



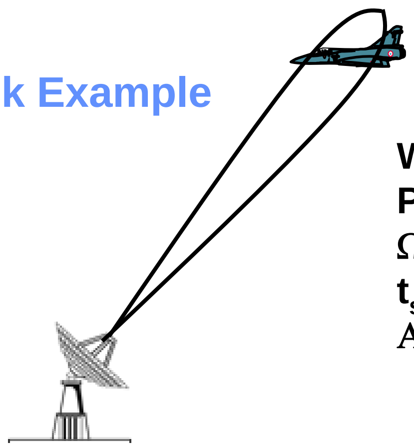
Track & Search Radar Range Equations

Track Radar Equation

$$S/N = \frac{P_t\, G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k\, T_s\, B_n\, L}$$

- When the location of a target is known and the antenna is pointed toward the target.

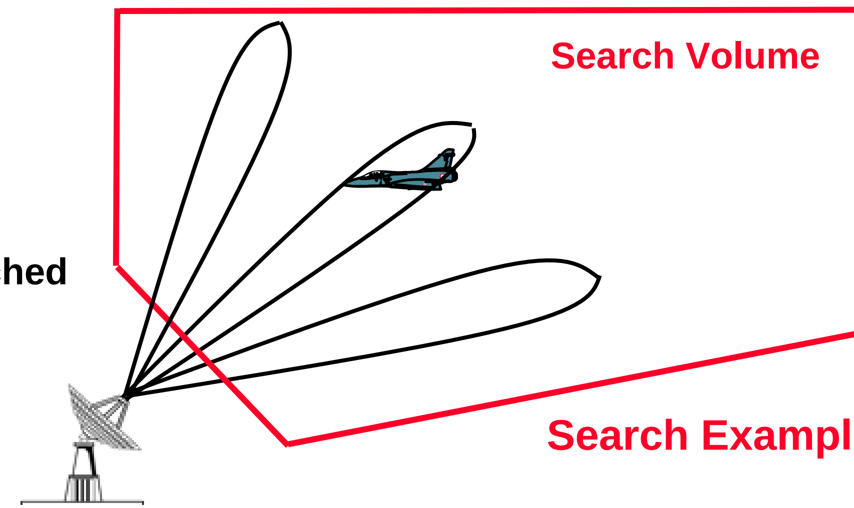
Track Example



Search Radar Equation

$$S/N = \frac{P_{av}\, A_e\, t_s\, \sigma}{4\pi\Omega\, R^4 k\, T_s\, L}$$

- When the target's location is unknown, and the radar has to search a large angular region to find it.



Search Example

Where:
P_{av} = average power
Ω = solid angle searched
t_s = scan time for Ω
A_e = antenna area

Search Radar Range Equation

$$S / N = \frac{P_{av}\, A_e\, t_s\, \sigma}{4\, \pi\, \Omega\, R^4\, k\, T_s\, L}$$

Re-write as:

f (design parameters) = g (performance parameters)

$$\frac{P_{av}\, A_e}{k T_s\, L} = \frac{4\, \pi\, \Omega\, R^4\, (S/N)}{\sigma\, t_s}$$

- Ω: Angular coverage
- R^4: Range coverage
- (S/N): Measurement quality
- t_s: Time required
- σ: Target size

Scaling of Radar Equation

$$\frac{S}{N} = \frac{P_{av} A_e t_s \sigma}{4\pi R^4 \Omega \; k T_s L} \quad \Rightarrow \quad P_{av} = \frac{4\pi R^4 \Omega k T_s L (S/N)}{A_e t_s \sigma}$$

- **Power required is:**
  - Independent of wavelength
  - A very strong function of R
  - A linear function of everything else

Example Radar Can Perform Search at 1000 km Range
How Might It Be Modified to Work at 2000 km ?

Solutions Increasing R by 3 dB (x 2) Can Be Achieved by:

1. Increasing P_{av} by 12 dB (x 16)

or 2. Increasing Diameter by 6 dB (A by 12 dB)

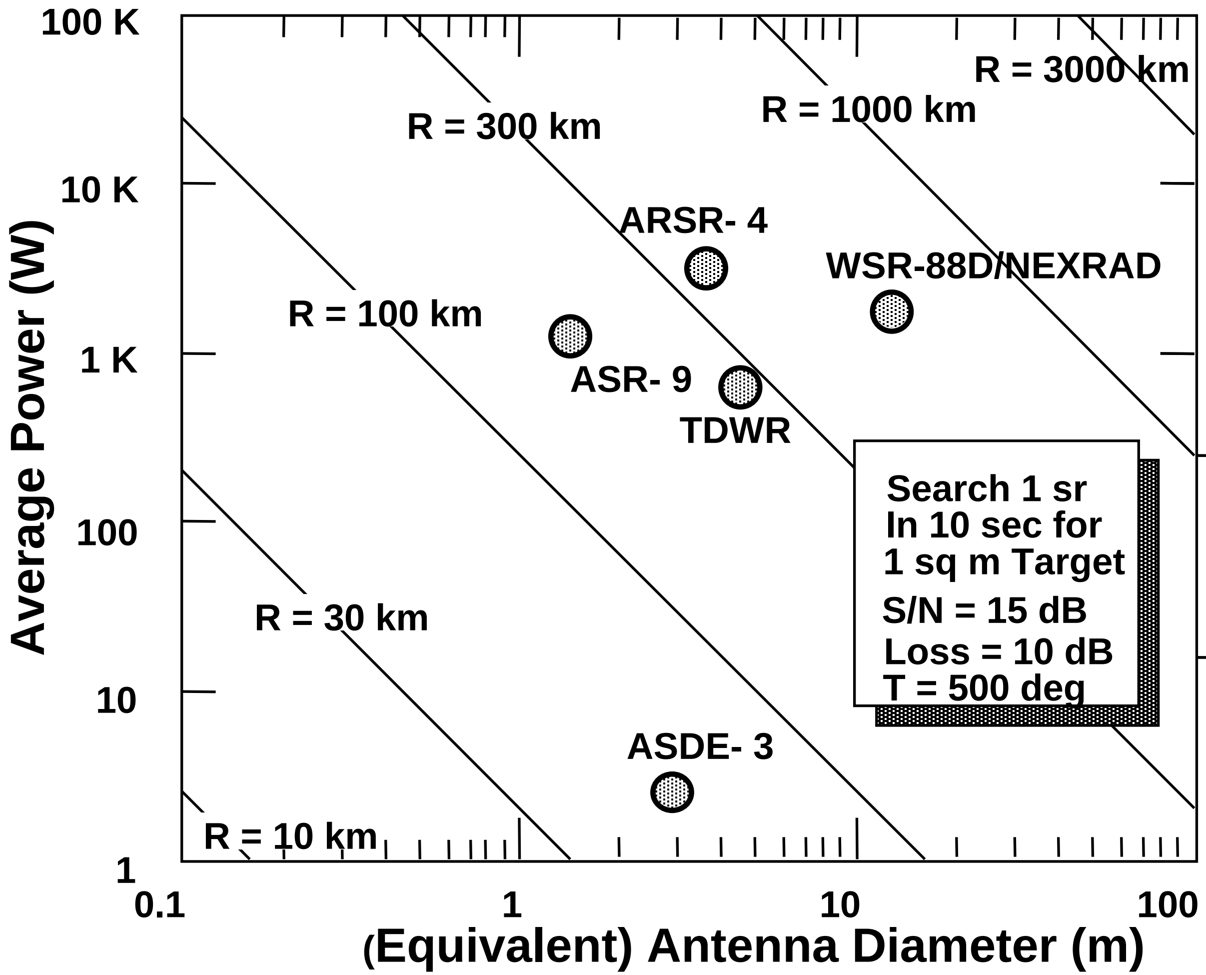
or 3. Increasing t_s by 12 dB

or 4. Decreasing Ω by 12 dB

or 5. Increasing σ by 12 dB

or 6. An Appropriate Combination of the Above

Search Radar Performance



ASR- 9
Airport Surveillance Radar

Courtesy of Northrop Grumman.
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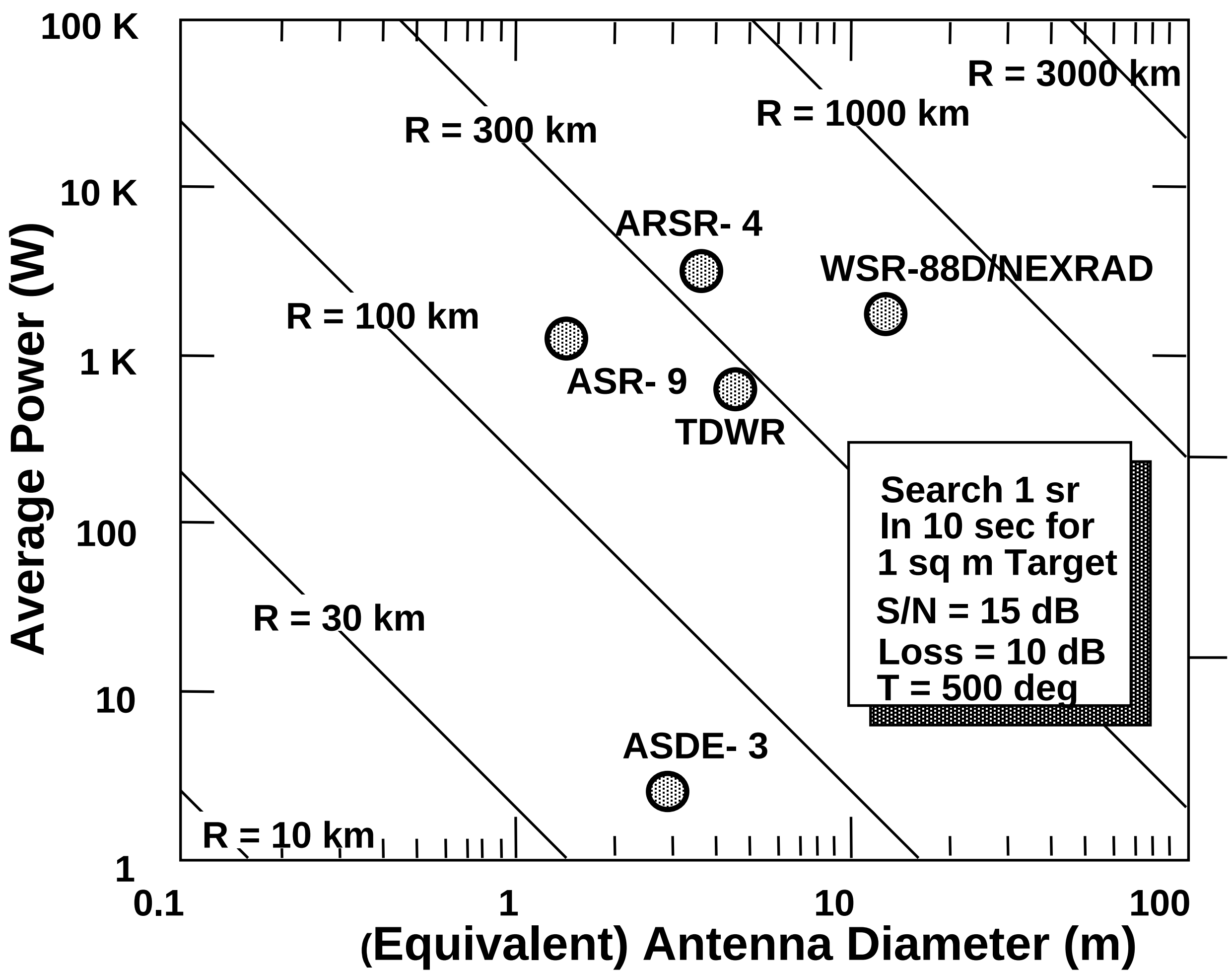
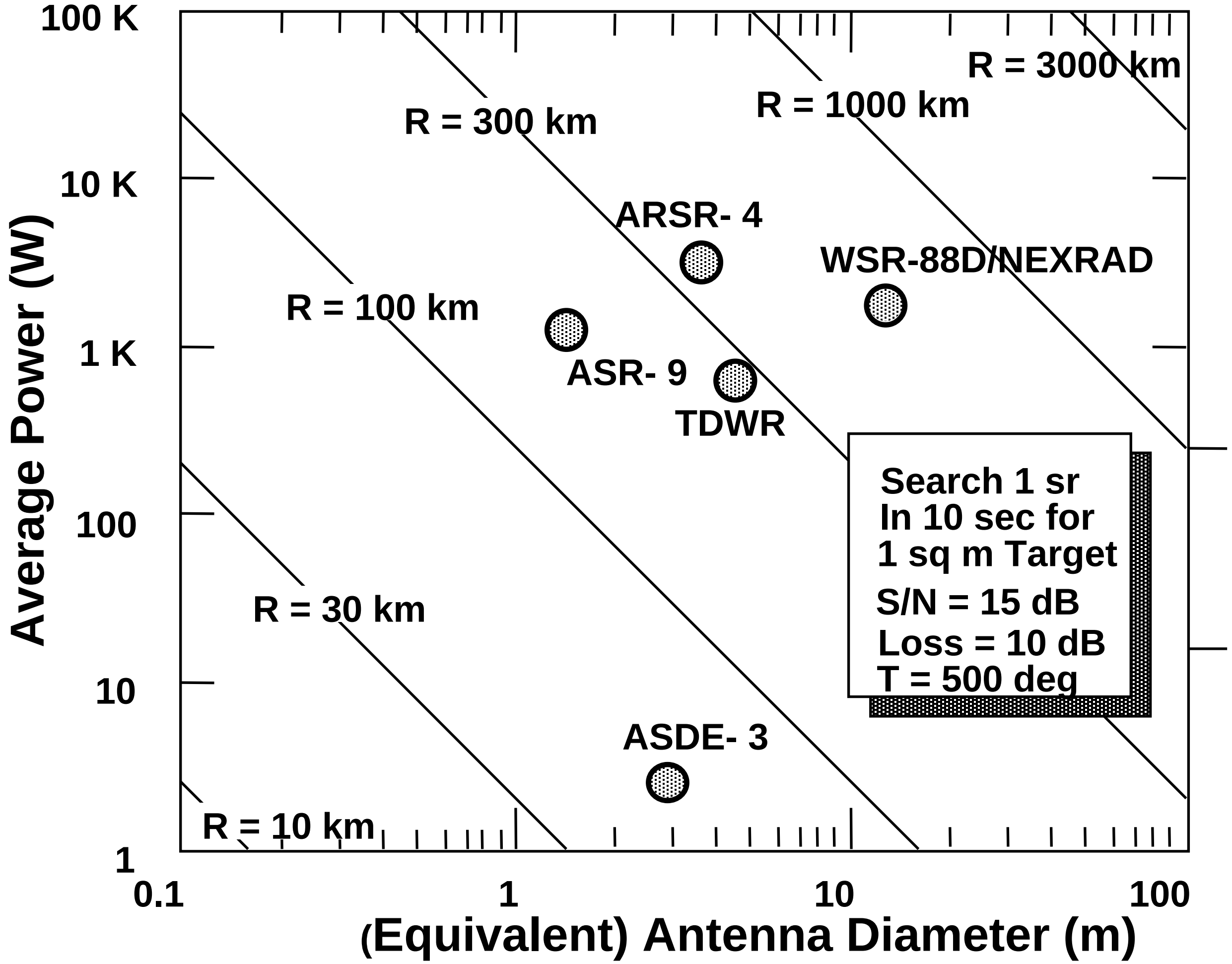
Search Radar Performance



ASDE- 3
Airport Surface Detection Equipment

Courtesy Lincoln Laboratory

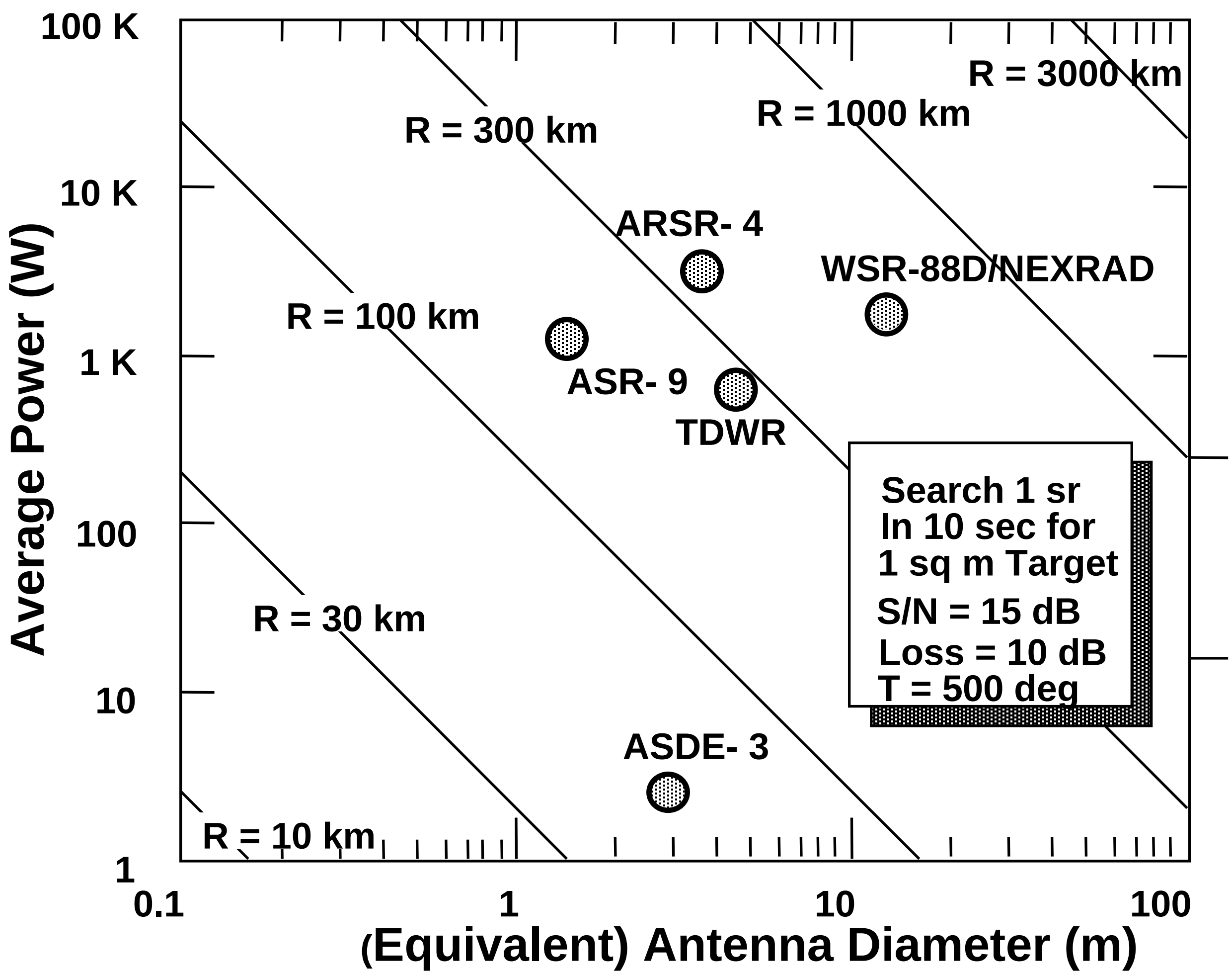
Search Radar Performance



ARSR- 4
Air Route Surveillance Radar

ARSR- 4 Antenna
(without Radome)

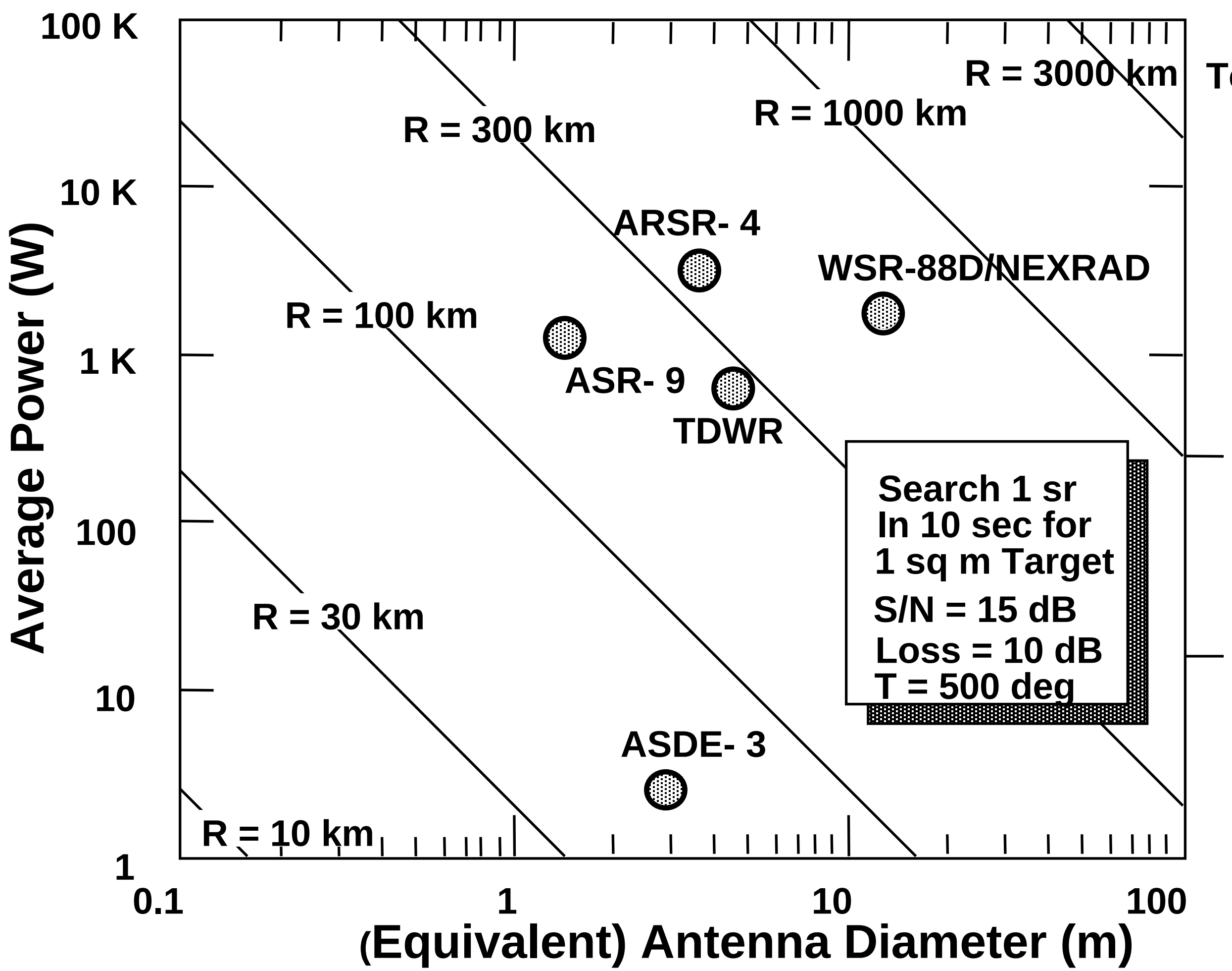
Search Radar Performance



WSR-88D / NEXRAD

Courtesy of NOAA.

Search Radar Performance



TDWR
Terminal Doppler Weather Radar

Courtesy of Raytheon.
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Loss Terms for Radar Equation

Transmit Losses

Radome
Waveguide Feed
Waveguide
Circulator
Low Pass Filters
Rotary Joints
Antenna Efficiency
Beam Shape
Scanning
Quantization
Atmospheric
Field Degradation

Receive Losses

Radome
Waveguide Feed
Waveguide
Combiner
Rotary Joints
Receiver Protector
Transmit / Receive Switch
Antenna Efficiency
Beam Shape
Scanning
Quantization
Weighting
Non-Ideal Filter
Doppler Straddling
Range Straddling
CFAR
Atmospheric
Field Degradation

Examples of Losses in Radar Equation

- **Beam Shape Loss**
  - Radar return from target with scanning radar is modulated by shape of antenna beam as it scans across target. Can be 2 to 4 dB
- **Scanning Antenna Loss**
  - For phased array antenna, gain of beam off boresight less than that on boresight
- **Plumbing Losses**
  - Transmit waveguide losses
  - Rotary joints, circulator, duplexer
- **Signal Processing Loss**
  - A /D Quantization Losses
  - Adaptive thresholding (CFAR) Loss
  - Range straddling Loss
  - Range and Doppler Weighting

Examples of Losses in Radar Equation

- **Atmospheric Attenuation Loss**
  - Radar beam attenuates as it travels through atmosphere (2 way loss)
- **Integration Loss**
  - Non coherent integration of pulses not as efficient as coherent integration
- **Margin (Field Degradation) Loss**
  - Characteristics of radar deteriorates over time.(3 dB not unreasonable
    - Water in transmission lines
    - Deterioration in receiver noise figure
    - Weak or poorly tuned transmitter tubes

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Example - Airport Surveillance Radar

- **Problem : Show that a radar with the parameters listed below, will get a reasonable S / N on an small aircraft at 60 nmi.**

Radar Parameters

Range	60 nmi
Aircraft cross section	1 m^2
Peak Power	1.4 Megawatts
Duty Cycle	0.000525
Pulsewidth	.6 microseconds
Bandwidth	1.67 MHz
Frequency	2800 MHz
Antenna Rotation Rare	12.8 RPM
Pulse Repetition Rate	1200 Hz
Antenna Size	4.9 m wide by 2.7 m high
Azimuth Beamwidth	1.35 ^{o}
System Noise Temp.	950 ^{o} K

$\lambda =$ c / f = .103 m

G = 4 π A / λ^2 = 15670 m^2
= 42 dB, (actually 33 dB with beam shaping losses)

Number of pulses per beamwidth = 21

Assume Losses = 8dB

Example - Airport Surveillance Radar

$$S / N = \frac{P_t \, G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k \, T_s \, B_n \, L}$$

P_t = 1.4 Megawatts	R = 111, 000 m
G = 33 dB = 2000	$T_s = 950\ ^oK$
$\lambda = .1$ m	B_n = 1.67 MHz
$\sigma = 1\ m^2$	L = 8dB = 6.3
$k = 1.38 \times 10^{-23}$ w / Hz ^{o}K	$(4\pi)^3 = 1984$

$$\frac{(1.4 \times 10^6 \text{ w})(2000)(2000)(.1\text{m})(.1\text{m})(1\text{m}^2)}{(1984)\,(1.11 \text{ X } 10^5 \text{ m})^4 (1.38 \times 10^{-23} \text{ w / Hz } ^o\text{K})\,(950\ ^o\text{K})\,(6.3)\,(1.67 \times 10^6 \text{ Hz})}$$

$$\frac{5.6 \times 10^{+6+3+3-1-1}}{415 \times 10^{+3+20-23+2+6}} = \frac{5.6 \times 10^{+10}}{4.15 \times 10^{+2+3+20-23+2+6}} = \frac{5.6 \times 10^{+10}}{4.15 \times 10^{+10}} = 1.35 = 1.3 \text{ dB}$$

S / N = 1.3 dB per pulse (21 pulses integrated) => S / N per dwell = 14.5 dB
+ 13.2 dB

Example - Airport Surveillance Radar

dB Method

		(+)	(-)
Peak Power	1.4 MW	61.5	
(Gain) ^{2}	33 db	66	
(Wavelength) ^{2}	.1 m		20
Cross section	1 m^2	0	
$(4\pi)^3$	1984		33
(Range) ^{4}	111 km		201.8
k	1.38×10^{-23} w / Hz ^{o} K	228.6	
System temp	950		29.8
Losses	8 dB		8
Bandwidth	1.67 MHz		62.2
		+ 356.1	- 354.8

+ 1.3 dB

S / N = 1.3 dB per pulse (21 pulses integrated) => S / N per dwell = 14.5 dB
(+ 13.2 dB)

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Cautions in Using the Radar Equation (1)

- The radar equation is simple enough that everybody can learn to use it

- The radar equation is complicated enough that anybody can mess it up if you are not careful (see next VG)

Cautions in Using the Radar Equation (2)

The Sanity Check

Take a Candidate Radar Equation

$$\frac{P\,A^2}{\lambda^2\,kT_s\,L} = \frac{4\,\pi\,R^4\,(S/N)}{\sigma\,t_t}$$

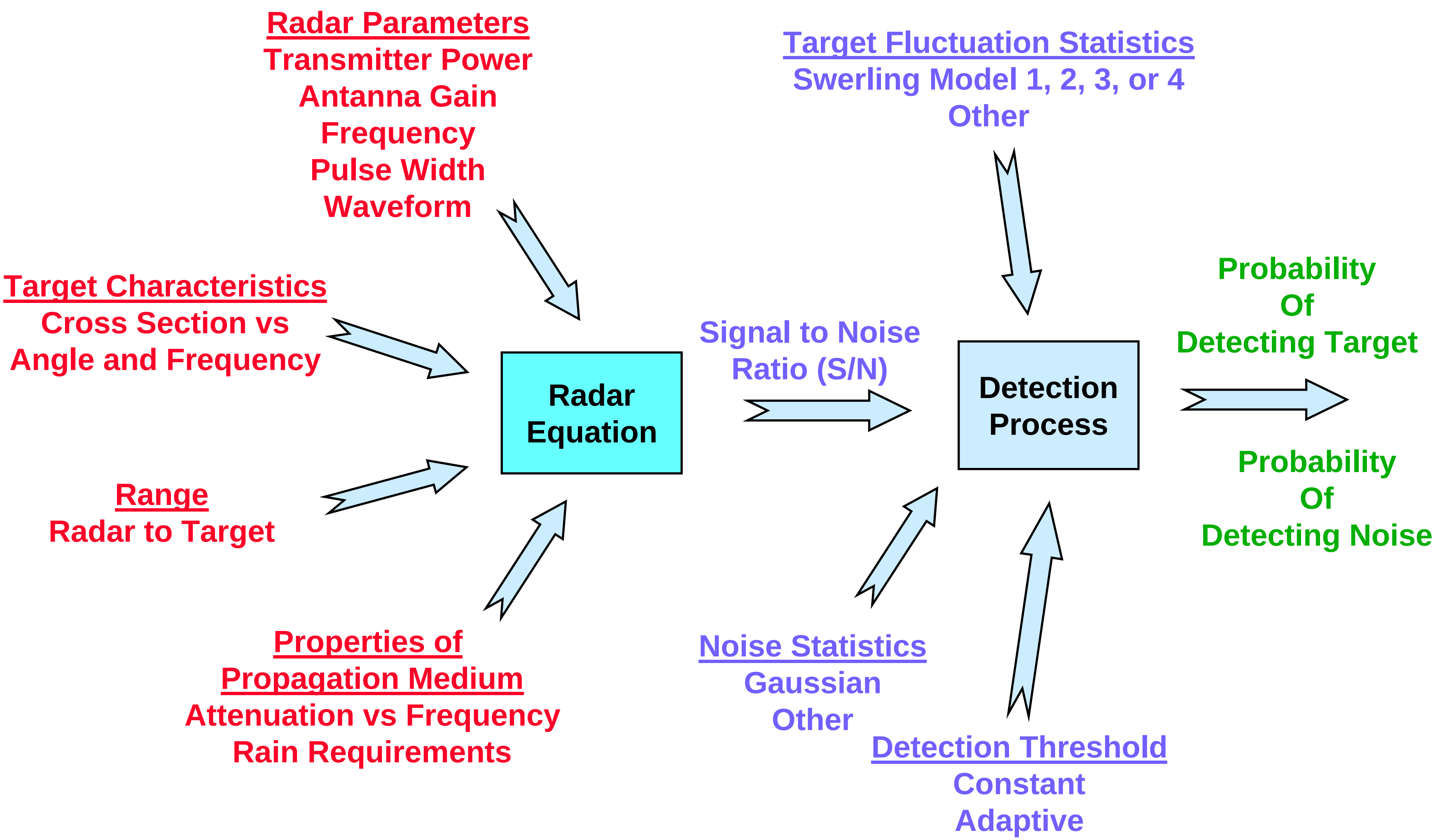
Check it Dimensionally

- P is energy/time
- kT_s is energy
- A and σ are distance squared
- λ and R are distance
- t_t is time
- S/N, L and 4π are dimensionless

Check if Dependencies Make Sense

– Increasing Range and S/N make requirements tougher
– Decreasing σ and t_t makes requirements tougher
– Increasing P and A make radar more capable
– Decreasing Noise Temp and Loss make radar more capable
– Decreasing λ makes radar more capable

Radar Equation and Detection Process



Summary

- The radar equation provides a simple connection between radar performance parameters and radar design parameters
- There are different radar equations for different radar functions
- Scaling of the radar equation lets you get a feeling for how the radar design might change to accommodate changing requirements
- Combination of the radar equation with cost or other constraints permits quick identification of critical radar design issues
- Be careful if the radar equation leads to unexpected results
  - Do a sanity check
  - Look for hidden variables or constraints
  - Try to compare parameters with those of a real radar

References

- Skolnik, M., Introduction to Radar Systems, New York, McGraw-Hill, 3rd Edition, 2001
- Barton, D. K., Modern radar System Analysis, Norwood, Mass., Artech House, 1988