Introduction to Radar Systems

The Radar Equation
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Introduction – The Radar Range Equation

The Radar Range Equation Connects:

1. **Target** Properties - e.g. Target Reflectivity (radar cross section)
2. **Radar** Characteristics - e.g. Transmitter Power, Antenna Aperture
3. Distance between **Target** and **Radar** - e.g. Range
4. Properties of the **Medium** - e.g. Atmospheric Attenuation.
Outline

• Introduction
• Introduction to Radar Equation
• Surveillance Form of Radar Equation
• Radar Losses
• Example
• Summary
Radar Range Equation

Power density from uniformly radiating antenna transmitting spherical wave

\[
\frac{P_t}{4\pi R^2}
\]

\(P_t = \) peak transmitter power
\(R = \) distance from radar
Radar Range Equation (continued)

Power density from isotropic antenna

\[ \frac{P_t}{4 \pi R^2} \]

Pt = peak transmitter power
R = distance from radar

Power density from directive antenna

\[ \frac{P_t G_t}{4 \pi R^2} \]

Gt = transmit gain

Gain is the radiation intensity of the antenna in a given direction over that of an isotropic (uniformly radiating) source

Gain = \[ 4 \pi \frac{A}{\lambda^2} \]
**Definition of Radar Cross Section (RCS or \( \sigma \))**

Radar Cross Section (RCS or \( \sigma \)) is a measure of the energy that a radar target intercepts and scatters back toward the radar.

\[
\sigma = \text{radar cross section units (meters)}^2
\]

Power of reflected signal at target:
\[
\frac{P_t G_t \sigma}{4\pi R^2}
\]

Power density of reflected signal at the radar:
\[
\frac{P_t G_t}{4\pi R^2} \quad \frac{\sigma}{4\pi R^2}
\]

Power density of reflected signal falls off as \((1/R^2)\)
Radar Range Equation (continued)

Power density of reflected signal at radar

\[ \frac{P_t G_t}{4 \pi R^2} \]  
\[ \frac{\sigma}{4 \pi R^2} \]

The received power = the power density at the radar times the area of the receiving antenna

\[ P_r = \frac{P_t G_t}{4 \pi R^2} \frac{\sigma A_e}{4 \pi R^2} \]

\( P_r \) = power received
\( A_e \) = effective area of receiving antenna

\[ \text{Power of reflected signal from target and received by radar} \]
Sources of Noise Received by Radar

- The total effect of these noise sources is represented by a single noise source at the antenna output terminal.

- The noise power at the receiver is given by: \( N = k B_n T_s \)

Noise from Many Sources Competes with the Target Echo

\[ k = \text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ joules / deg } ^{o} \text{K} \]
\[ T_s = \text{System Noise Temperature} \]
\[ B_n = \text{Noise bandwidth of receiver} \]
Radar Range Equation (continued)

Signal Power reflected from target and received by radar

\[ P_r = \frac{P_t G_t}{4\pi R^2} \frac{\sigma A_e}{4\pi R^2} \]

Average Noise Power

\[ N = k T_s B_n \]

Signal to Noise Ratio

\[ \frac{S}{N} = \frac{P_r}{N} \]

Assumptions:

- \( G_t = G_r \)
- \( L = \) Total System Losses
- \( T_0 = 290^\circ K \)

Signal to Noise Ratio (S/N or SNR) is the standard measure of a radar’s ability to detect a given target at a given range from the radar

“S/N = 13 dB on a 1 m\(^2\) target at a range of 1000 km”

radar cross section of target
System Noise Temperature

The System Noise Temperature, $T_S$, is divided into 3 components:

$$T_S = T_a + T_r + L_r T_e$$

- $T_a$ is the contribution from the antenna
  - Apparent temperature of sky (from graph)
  - Loss within antenna
- $T_r$ is the contribution from the RF components between the antenna and the receiver
  - Temperature of RF components
- $L_r$ is the loss of input RF components
- $T_e$ is the temperature of the receiver
  - Noise factor of receiver
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Track Radar Range Equation

\[
\frac{S}{N} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_s B_n L}
\]

- When the location of a target is known and the antenna is pointed toward the target.

Track Example
Track & Search Radar Range Equations

Track Radar Equation

\[
\frac{S}{N} = \frac{P_t G^2 \lambda^2 \sigma}{(4 \pi)^3 R^4 k T_s B_n L}
\]

Where:
- \( P_t \) = average power
- \( \Omega \) = solid angle searched
- \( t_s \) = scan time for \( \Omega \)
- \( A_e \) = antenna area

Search Radar Equation

\[
\frac{S}{N} = \frac{P_{av} A_e t_s \sigma}{4 \pi \Omega R^4 k T_s L}
\]

- When the location of a target is known and the antenna is pointed toward the target.
- When the target's location is unknown, and the radar has to search a large angular region to find it.
Search Radar Range Equation

\[
\frac{S}{N} = \frac{P_{av} A_e t_s \sigma}{4 \pi \Omega R^4 k T_s L}
\]

Re-write as:

\[f(\text{design parameters}) = g(\text{performance parameters})\]

\[
\frac{P_{av} A_e}{k T_s L} = \frac{4 \pi \Omega R^4 (S/N) \sigma t_s}{\sigma t_s}
\]

Angular coverage

Range coverage

Measurement quality

Time required

Target size
Scaling of Radar Equation

\[ \frac{S}{N} = \frac{P_{av} A_e t_s \sigma}{4\pi R^4 \Omega k T_s L} \quad \Rightarrow \quad P_{av} = \frac{4\pi R^4 \Omega k T_s L (S/N)}{A_e t_s \sigma} \]

- Power required is:
  - Independent of wavelength
  - A very strong function of R
  - A linear function of everything else

**Example**
Radar Can Perform Search at 1000 km Range
How Might It Be Modified to Work at 2000 km?

**Solutions**
Increasing R by 3 dB (x 2) Can Be Achieved by:

1. Increasing \( P_{av} \) by 12 dB (x 16)
2. Increasing Diameter by 6 dB (\( A \) by 12 dB)
3. Increasing \( t_s \) by 12 dB
4. Decreasing \( \Omega \) by 12 dB
5. Increasing \( \sigma \) by 12 dB
6. An Appropriate Combination of the Above
Search Radar Performance

- **Average Power (W)**
  - 100 K
  - 10 K
  - 1 K
  - 100

- **(Equivalent) Antenna Diameter (m)**
  - R = 10 km
  - R = 30 km
  - R = 100 km
  - R = 300 km
  - R = 1000 km
  - R = 3000 km

- **Search 1 sr In 10 sec for 1 sq m Target**
  - S/N = 15 dB
  - Loss = 10 dB
  - T = 500 deg

- **ASR-9**
  - Airport Surveillance Radar

- **ASR-4**, **WSR-88D/NEXRAD**, **TDWR**, **ASDE-3**

- **Courtesy of Northrop Grumman. Used with permission.**
Search Radar Performance

(Equivalent) Antenna Diameter (m) vs. Average Power (W)

- **ASDE-3**
- **ARSR-4**
- **ASR-9**
- **TDWR**
- **WSR-88D/NEXRAD**

Search 1 sr in 10 sec for 1 sq m Target
S/N = 15 dB
Loss = 10 dB
T = 500 deg

Search Radar Performance

- **ASDE-3**
- **TDWR**

Airport Surface Detection Equipment

Courtesy Lincoln Laboratory
Search Radar Performance

- **Average Power (W)**
  - 100 K
  - 10 K
  - 1 K
  - 100

- **(Equivalent) Antenna Diameter (m)**
  - R = 100 km
  - R = 30 km
  - R = 10 km
  - R = 300 km
  - R = 1000 km
  - R = 3000 km

- **Search Radar Performance**
  - Search 1 sr
  - In 10 sec for
  - 1 sq m Target
  - S/N = 15 dB
  - Loss = 10 dB
  - T = 500 deg

- **Radar Systems**
  - ARSR- 4
  - ASR- 9
  - ASDE- 3
  - WSR-88D/NEXRAD
  - TDWR

- **Antenna Details**
  - ARSR- 4 Antenna (without Radome)
  - Courtesy of Northrop Grumman.
  - Used with permission.
**Search Radar Performance**

![Graph showing search radar performance with labels and markers for different systems and distances, including ASR-9, ASDE-3, TDWR, ARSR-4, WSR-88D/NEXRAD, and R values for 10 km, 30 km, 100 km, 300 km, 1000 km, and 3000 km.]

- **Average Power (W):** 100 K, 10 K, 1 K, 100
- **(Equivalent) Antenna Diameter (m):** R = 10 km, R = 30 km, R = 100 km, R = 300 km, R = 1000 km, R = 3000 km

- **WSR-88D / NEXRAD**
  - Courtesy of NOAA.

**Search 1 sr In 10 sec for 1 sq m Target**
- S/N = 15 dB
- Loss = 10 dB
- T = 500 deg
Search Radar Performance

![Diagram showing search radar performance with points marking different radars at various radii and average power.](image)

- **ARSR-4**
- **ASR-9**
- **WSR-88D/NEXRAD**
- **ASDE-3**
- **TDWR**

**Search 1 sr In 10 sec for 1 sq m Target**
- S/N = 15 dB
- Loss = 10 dB
- T = 500 deg

**Terminal Doppler Weather Radar**

*Courtesy of Raytheon. Used with permission.*
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## Loss Terms for Radar Equation

<table>
<thead>
<tr>
<th>Transmit Losses</th>
<th>Receive Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radome</td>
<td>Radome</td>
</tr>
<tr>
<td>Waveguide Feed</td>
<td>Waveguide Feed</td>
</tr>
<tr>
<td>Waveguide</td>
<td>Waveguide</td>
</tr>
<tr>
<td>Circulator</td>
<td>Combiner</td>
</tr>
<tr>
<td>Low Pass Filters</td>
<td>Rotary Joints</td>
</tr>
<tr>
<td>Rotary Joints</td>
<td>Receiver Protector</td>
</tr>
<tr>
<td>Antenna Efficiency</td>
<td>Transmit / Receive Switch</td>
</tr>
<tr>
<td>Beam Shape</td>
<td>Antenna Efficiency</td>
</tr>
<tr>
<td>Scanning</td>
<td>Beam Shape</td>
</tr>
<tr>
<td>Quantization</td>
<td>Scanning</td>
</tr>
<tr>
<td>Atmospheric</td>
<td>Quantization</td>
</tr>
<tr>
<td>Field Degradation</td>
<td>Weighting</td>
</tr>
<tr>
<td></td>
<td>Non-Ideal Filter</td>
</tr>
<tr>
<td></td>
<td>Doppler Straddling</td>
</tr>
<tr>
<td></td>
<td>Range Straddling</td>
</tr>
<tr>
<td></td>
<td>CFAR</td>
</tr>
<tr>
<td></td>
<td>Atmospheric</td>
</tr>
<tr>
<td></td>
<td>Field Degradation</td>
</tr>
</tbody>
</table>

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Examples of Losses in Radar Equation

- **Beam Shape Loss**
  - Radar return from target with scanning radar is modulated by shape of antenna beam as it scans across target. Can be 2 to 4 dB

- **Scanning Antenna Loss**
  - For phased array antenna, gain of beam off boresight less than that on boresight

- **Plumbing Losses**
  - Transmit waveguide losses
  - Rotary joints, circulator, duplexer

- **Signal Processing Loss**
  - A/D Quantization Losses
  - Adaptive thresholding (CFAR) Loss
  - Range straddling Loss
  - Range and Doppler Weighting
Examples of Losses in Radar Equation

• Atmospheric Attenuation Loss
  – Radar beam attenuates as it travels through atmosphere (2 way loss)

• Integration Loss
  – Non coherent integration of pulses not as efficient as coherent integration

• Margin (Field Degradation) Loss
  – Characteristics of radar deteriorates over time.(3 dB not unreasonable
    • Water in transmission lines
    • Deterioration in receiver noise figure
    • Weak or poorly tuned transmitter tubes
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Example - Airport Surveillance Radar

• Problem: Show that a radar with the parameters listed below, will get a reasonable S/N on an small aircraft at 60 nmi.

Radar Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>60 nmi</td>
</tr>
<tr>
<td>Aircraft cross section</td>
<td>1 m²</td>
</tr>
<tr>
<td>Peak Power</td>
<td>1.4 Megawatts</td>
</tr>
<tr>
<td>Duty Cycle</td>
<td>0.000525</td>
</tr>
<tr>
<td>Pulsewidth</td>
<td>.6 microseconds</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>1.67 MHz</td>
</tr>
<tr>
<td>Frequency</td>
<td>2800 MHz</td>
</tr>
<tr>
<td>Antenna Rotation Rate</td>
<td>12.8 RPM</td>
</tr>
<tr>
<td>Pulse Repetition Rate</td>
<td>1200 Hz</td>
</tr>
<tr>
<td>Antenna Size</td>
<td>4.9 m wide by 2.7 m high</td>
</tr>
<tr>
<td>Azimuth Beamwidth</td>
<td>1.35 °</td>
</tr>
<tr>
<td>System Noise Temp.</td>
<td>950 ° K</td>
</tr>
</tbody>
</table>

$$\lambda = \frac{c}{f} = 0.103 \text{ m}$$

$$G = 4 \pi \frac{A}{\lambda^2} = 15670 \text{ m}^2$$

= 42 dB, (actually 33 dB with beam shaping losses)

Number of pulses per beamwidth = 21

Assume Losses = 8dB
Example - Airport Surveillance Radar

\[
S / N = \frac{P_t G^2 \lambda^2 \sigma}{(4 \pi)^3 R^4 k T_s B_n L}
\]

- \(P_t = 1.4 \text{ Megawatts}\)
- \(G = 33 \text{ dB} = 2000\)
- \(\lambda = 0.1 \text{ m}\)
- \(\sigma = 1 \text{ m}^2\)
- \(k = 1.38 \times 10^{-23} \text{ w/Hz}^\circ \text{K}\)
- \(R = 111,000 \text{ m}\)
- \(T_s = 950^\circ \text{K}\)
- \(B_n = 1.67 \text{ MHz}\)
- \(L = 8 \text{ dB} = 6.3\)

\[
\frac{5.6 \times 10^{+6+3+3-1-1}}{415 \times 10^{+3+20-23+2+6}} = \frac{5.6 \times 10^{+10}}{4.15 \times 10^{+10}} = 1.35 = 1.3 \text{ dB}
\]

\(S / N = 1.3 \text{ dB per pulse (21 pulses integrated)} \Rightarrow S / N \text{ per dwell} = 14.5 \text{ dB} + 13.2 \text{ dB}\)
### Example - Airport Surveillance Radar

**dB Method**

<table>
<thead>
<tr>
<th>(+)</th>
<th>(-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Power</td>
<td>1.4 MW</td>
</tr>
<tr>
<td>Gain</td>
<td>33 dB</td>
</tr>
<tr>
<td>Wavelength</td>
<td>2.1 m</td>
</tr>
<tr>
<td>Cross section</td>
<td>1 m²</td>
</tr>
<tr>
<td>(4\pi)^3</td>
<td>1984</td>
</tr>
<tr>
<td>Range</td>
<td>111 km</td>
</tr>
<tr>
<td>k</td>
<td>1.38 x 10^{-23} w / Hz ° K</td>
</tr>
<tr>
<td>System temp</td>
<td>950</td>
</tr>
<tr>
<td>Losses</td>
<td>8 dB</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>1.67 MHz</td>
</tr>
<tr>
<td>+ 356.1</td>
<td>- 354.8</td>
</tr>
<tr>
<td>+ 1.3 dB</td>
<td></td>
</tr>
</tbody>
</table>

\[S / N = 1.3 \text{ dB per pulse (21 pulses integrated)} \Rightarrow S / N \text{ per dwell} = 14.5 \text{ dB (} + 13.2 \text{ dB)}\]
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Cautions in Using the Radar Equation (1)

• The radar equation is simple enough that everybody can learn to use it

• The radar equation is complicated enough that anybody can mess it up if you are not careful (see next VG)
Cautions in Using the Radar Equation (2)

The Sanity Check

Take a Candidate Radar Equation

\[
\frac{PA^2}{\lambda^2 kT_s L} = \frac{4\pi R^4 (S/N)}{\sigma t_t}
\]

Check it Dimensionally

- \( P \) is energy/time
- \( kT_s \) is energy
- \( A \) and \( \sigma \) are distance squared
- \( \lambda \) and \( R \) are distance
- \( t_t \) is time
- \( S/N, L \) and \( 4\pi \) are dimensionless

Check if Dependencies Make Sense

- Increasing Range and S/N make requirements tougher
- Decreasing \( \sigma \) and \( t_t \) makes requirements tougher
- Increasing \( P \) and \( A \) make radar more capable
- Decreasing Noise Temp and Loss make radar more capable
- Decreasing \( \lambda \) makes radar more capable
Radar Equation and Detection Process

Radar Parameters
- Transmitter Power
- Antenna Gain
- Frequency
- Pulse Width
- Waveform

Target Characteristics
- Cross Section vs Angle and Frequency

Range
- Radar to Target

Properties of Propagation Medium
- Attenuation vs Frequency
- Rain Requirements

Target Fluctuation Statistics
- Swerling Model 1, 2, 3, or 4
- Other

Probability of Detecting Target

Signal to Noise Ratio (S/N)

Detection Threshold
- Constant
- Adaptive

Probability of Detecting Noise

Noise Statistics
- Gaussian
- Other
Summary

• The radar equation provides a simple connection between radar performance parameters and radar design parameters

• There are different radar equations for different radar functions

• Scaling of the radar equation lets you get a feeling for how the radar design might change to accommodate changing requirements

• Combination of the radar equation with cost or other constraints permits quick identification of critical radar design issues

• Be careful if the radar equation leads to unexpected results
  – Do a sanity check
  – Look for hidden variables or constraints
  – Try to compare parameters with those of a real radar
References