

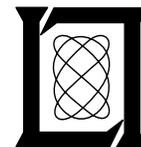
**Project Report
ATC-380**

Triangle TCAS Antenna

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31 March 2011

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16. Abstract The Traffic Alert and Collision Avoidance (TCAS) provides a pilot display showing the range and bearing of nearby aircraft. TCAS obtains the bearing information by using an angle-of-arrival antenna. In the development of TCAS at Lincoln Laboratory, the first airborne tests were conducted using an Adcock antenna, which is a small square array of four monopole elements. This report describes an alternative antenna for TCAS, using three elements in the shape of a triangle. It is shown that the triangle antenna is less sensitive to receiver noise, and that improvement factor is about 10 dB.			
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The Traffic Alert and Collision Avoidance (TCAS) provides a display showing a pilot the range and bearing of nearby aircraft. TCAS obtains the bearing information by using an angle-of-arrival antenna. In the development of TCAS at Lincoln Laboratory, the first airborne tests were conducted using an Adcock antenna. It is a small square array of four elements. Figure 1 shows the outward appearance of an Adcock antenna consisting of four bare monopoles. Bare monopoles can be used on small aircraft that fly at relatively low speeds. For higher speed airliners, it is more appropriate to cover the four monopoles with a radome to reduce wind resistance and also to shorten the monopoles by top loading.

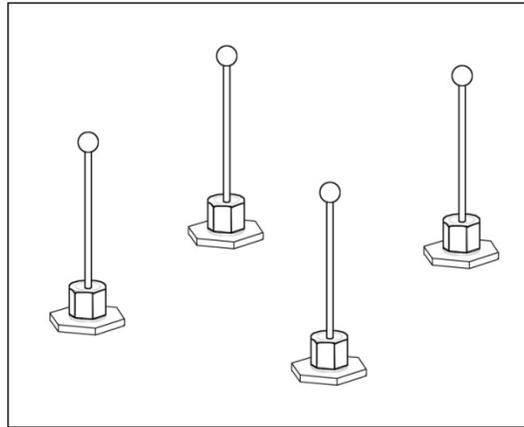


Figure 1. Adcock antenna of four bare monopoles.

The four receptions are fed to a phasing network, having two output ports. These outputs are fed to two receivers, which determine the phase between them. The resulting phase is used to estimate the angle-of-arrival of the signal being received (References 1 and 2).

This report describes an alternative antenna for TCAS, using three elements in the shape of a triangle. Figure 2 illustrates the concept of the triangle antenna as compared with the Adcock antenna. The triangle antenna offers an advantage in angle-of-arrival accuracy as affected by receiver noise. The amount of improvement is estimated in this document.

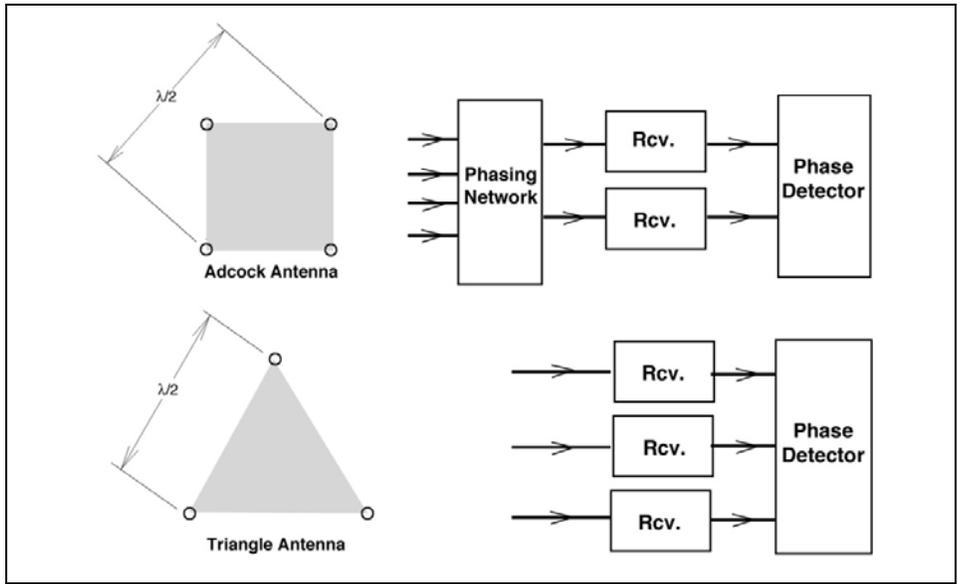


Figure 2. Comparison between Adcock antenna and triangle antenna.

λ = wavelength

The main performance difference between an Adcock antenna and the triangle antenna is illustrated in Figure 3. For the Adcock configuration, the measured phase varies nearly linearly with azimuth, changing by 360 degrees as azimuth varies over 360 degrees. So the rate of change is approximately

Slope = 1 radian of phase change per radian of azimuth (Adcock) .

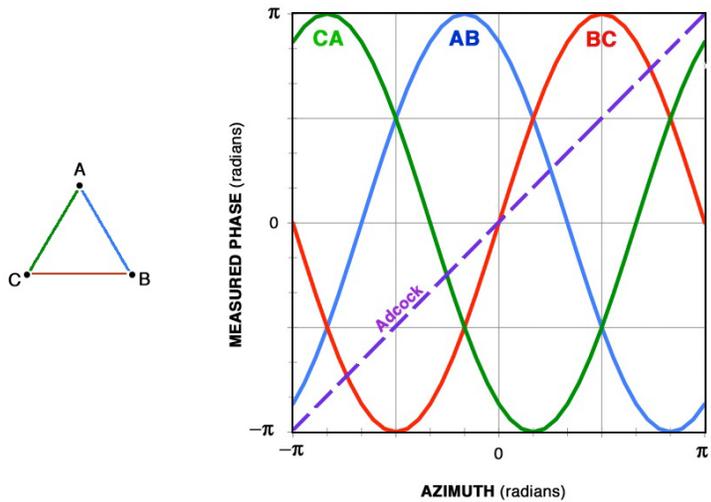


Figure 3. Measured phase as a function of azimuth for both the Adcock antenna and the triangle antenna.

For the triangle antenna, let A, B, and C refer to the three monopoles (beginning at the top going clockwise in Figure 3). Azimuth is defined as clockwise positive, beginning in the direction of A. Whereas the Adcock antenna has a single phase vs. azimuth characteristic, the triangle antenna has three, and in each curve the rate of change is substantially higher relative to the Adcock antenna.

Maximum slope = π radians of phase change per radian of azimuth (triangle)

This is the maximum value, but as shown below, when the three phases are used together, the effect is to get the benefit of the maximum slope in every azimuth direction. A higher slope is beneficial because an error in measured phase, which can be caused by receiver noise, translates into a smaller error in estimated azimuth.

Based on the substantial improvement in slope (measured phase vs. azimuth), we might simply conclude that the triangle antenna has superior azimuth accuracy by a factor of π . The analysis that follows provides more specifics, including the fact that each of the three receivers contributes small errors to two of the measured phases, so the resulting measured phases are statistically correlated. The material that follows also shows more specifically how the three measurements of phase can be used together to determine the azimuth angle-of-arrival.

ANALYSIS

The analysis in this document applies to receptions from a transmitter infinitely far away. Mutual coupling is not included in this analysis, so the different monopole receptions differ from each other only as a consequence of the different path lengths from the transmitter. Phase between two receptions is defined counterclockwise positive and negative when the second of the two monopoles is farther from the transmitter. For example, phase AB is negative when B is farther from the transmitter.

The use of three phase measurements to make an estimate of azimuth is illustrated in Figure 4. The three phase measurements can be considered to describe a curve in the three-dimensional coordinate system,

$$\begin{aligned}x &= \text{phase AB} \\y &= \text{phase BC} \\z &= \text{phase CA} .\end{aligned}$$

As azimuth varies, the locus of points in the x,y,z coordinate system is seen in Figure 4 to be a closed figure like an ellipse or circle. After rotating coordinates, it is found that the locus is a perfect circle in the $x''y''$ plane. There are two rotations, as follows. The first rotation is in the xy plane, by an angle $\theta_1 = \pi/4$. Denoting the new coordinates as x',y',z' , the second rotation is in the $x'z'$ plane, by an angle $\theta_2 = \arctan(-\sqrt{2})$. Putting the two rotation formulas together, the combined formulas for calculating x'' and y'' from the measurements x,y,z are as follows:

$$\begin{aligned}x'' &= x \cos \theta_1 \cos \theta_2 + y \sin \theta_1 \cos \theta_2 + z \sin \theta_2 \\y'' &= -x \sin \theta_1 + y \cos \theta_1\end{aligned}$$

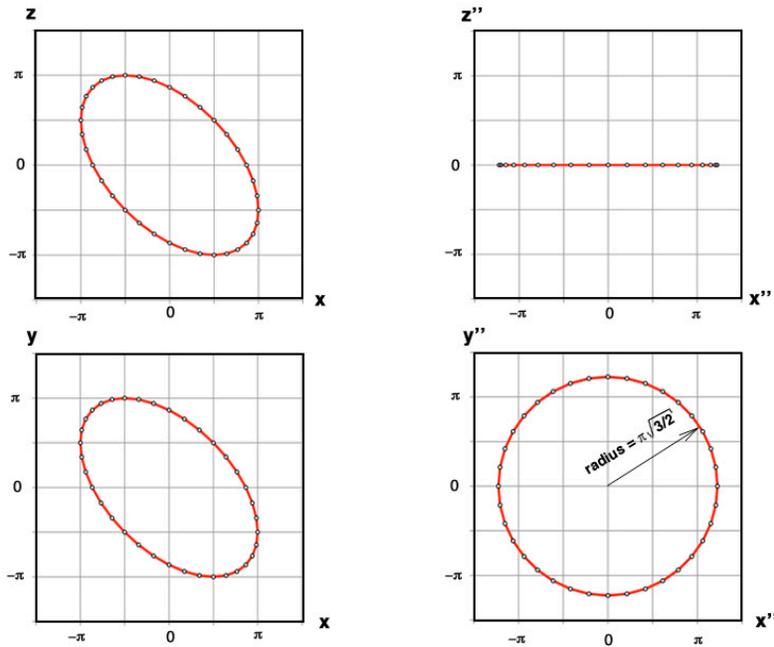


Figure 4. Use of three phase measurements to estimate azimuth.

The xyz coordinates on the left apply to the direct phase measurements.

The x''y''z'' coordinates on the right apply after two rotations.

The locus in the x''y'' plane is a circle of radius $\pi * \sqrt{3/2}$, which equals 3.848 radians. The discrete points in this figure are samples of azimuth over 360° evenly spaced by 10°. The even spacing that appears in the x''y'' plane suggests that the polar angle in the x''y'' plane can be taken as an estimate of azimuth, which in fact is true. Specifically, in the absence of noise,

$$\text{polar angle} = \arctan(x'', y'') = \text{azimuth} - \pi/6 ,$$

where polar angle is take to be zero in the plus x'' direction, counterclockwise positive. In other words, azimuth can be estimated by taking the polar angle in the x''y'' plane and adding 30 degrees.

Receiver Noise. Each of the receivers illustrated in Figure 2 will contribute noise, which will cause errors in the azimuth estimate. Analysis of the resulting azimuth accuracy begins by characterizing each receiver as adding a noise of power N to the reception. The receiver noises in different receivers are considered to be independent. The received signal power is denoted S, and therefore the signal-to-noise ratio in one receiver is S/N.

The analysis of receiver noise effects on azimuth accuracy is given in the Appendix. Mutual coupling between the monopoles is not included in the analysis. For cases of high S/N, the results are:

$$\begin{array}{lll} \text{mean square azimuth error} = & 1/(\pi^2 S/N) & \text{triangle antenna} \\ & 1/(S/N) & \text{Adcock antenna (linear model) .} \end{array}$$

This result for the Adcock antenna applies to the perfectly linear model illustrated in Figure 3, which is an idealization as described below. These results show that the triangle antenna improves azimuth accuracy by a factor of π^2 in mean square values. Expressed in dB, the improvement is $10 \log(\pi^2) = 9.9$ dB. Expressed in root-mean-square (rms) values, the improvement is a factor of π .

If for example S/N were 20 dB at each receiver, the rms errors attributable to receiver noise would be

$$\begin{array}{lll} \text{rms error} = & 0.03 \text{ radian} = 1.8^\circ & \text{triangle antenna} \\ & 0.1 \text{ radian} = 5.7^\circ & \text{Adcock antenna (linear model) .} \end{array}$$

ADCOCK RIPPLES

The perfectly linear model for the Adcock antenna is serviceable for a first-order calculation, but it is somewhat more accurate to calculate the angle transfer function from the formulas for combining the four monopole receptions into two channels. The combining network is illustrated in Figure 5 (from Reference 1, page 85).

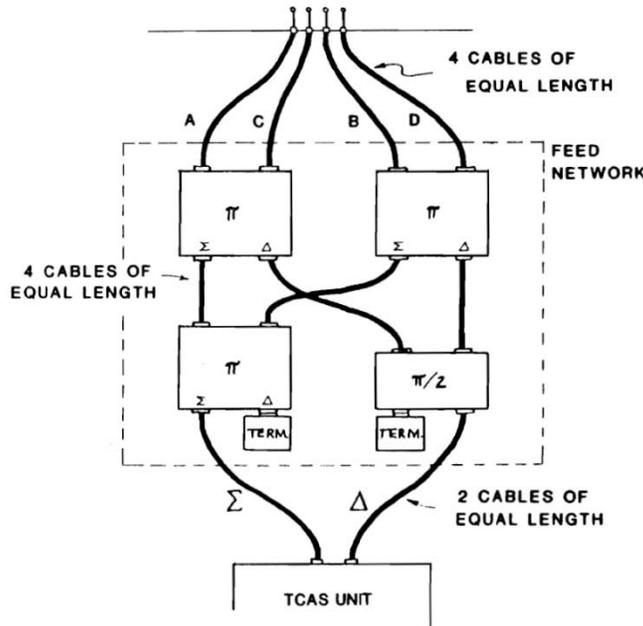


Figure 5. Adcock antenna combining network.

Apart from a constant phase offset associated with the interconnecting cables, the combining formulas are as follows:

$$\Sigma = (A + B + C + D)/2$$

$$\Delta = [(A - C) + j(B - D)]/2$$

When these formulas are evaluated for azimuth values over a full cycle, the transfer function appears as shown in Figure 6.

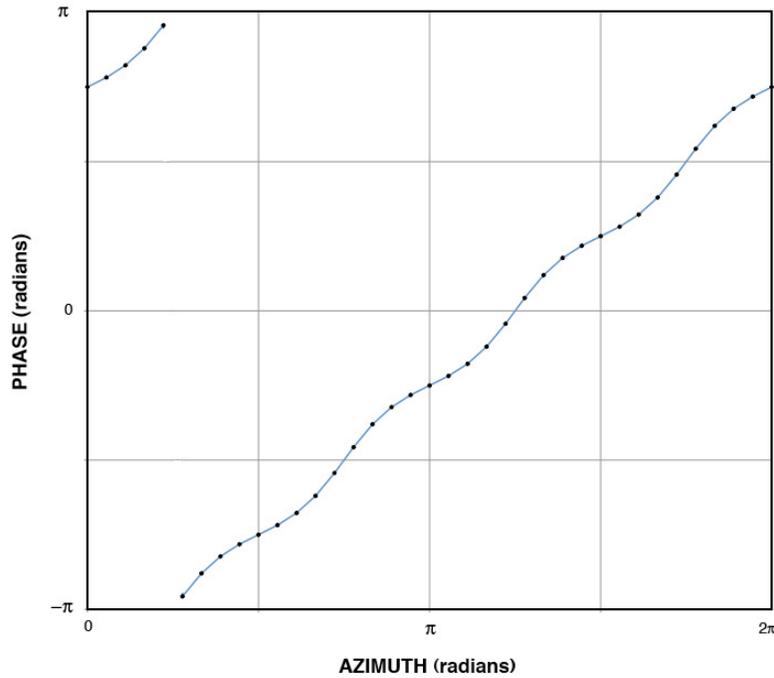


Figure 6. Transfer function of the Adcock antenna.

The transfer function is seen to be approximately linear over a full cycle, with an offset related to the interconnecting cables. Superimposed on the overall linear behavior are ripples having four cycles. If the TCAS uses a simple linear formula to estimate azimuth, these ripples will contribute errors of about ± 8 degrees. Also, because the slope of the transfer function is not constant, the sensitivity to receiver noise will be degraded in the directions where slope is less. The amount of slope change caused by the ripples is shown in Figure 7. Although the average slope is 1, as in the linear model, the ripples cause the slope to depart considerably from the average. The result is that accuracy as affected by receiver noise is degraded by about a factor of 2 in certain directions.

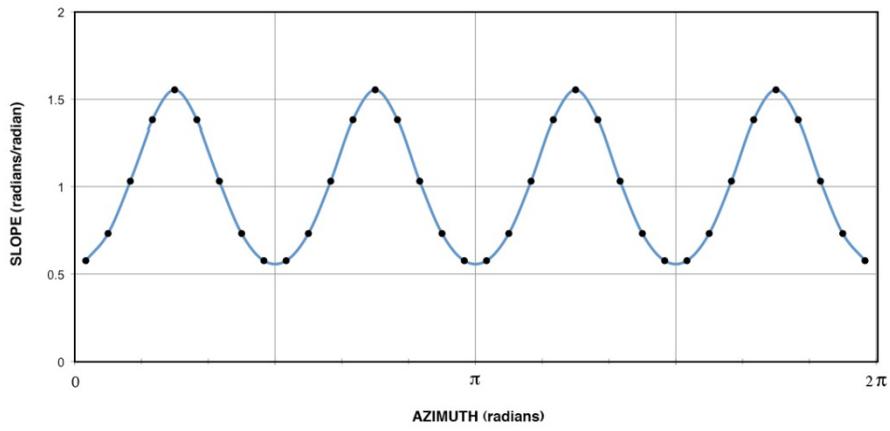


Figure 7. Slope of the Adcock transfer function.

DIMENSIONS OF THE TRIANGLE ANTENNA

As described previously, the spacing of the three monopoles is $\lambda/2$ on each side, where λ is the wavelength. That size is a natural choice because the cosine functions illustrated in Figure 3 vary regularly between plus and minus π . But the behavior is nearly the same if the spacing is somewhat less than $\lambda/2$. The only difference is that the cosine functions do not quite reach plus and minus π , and so the slope values are somewhat less and the accuracy is somewhat less. The possibility of having the physical size different from $\lambda/2$ makes it possible to use the triangle antenna over a band of frequencies, provided that a calibration is performed at each frequency. It is also possible to make the size somewhat larger than $\lambda/2$. When that's true, what was a circle in the x"y" plane becomes distorted by folding back instead of expanding. That too is a possible implementation, provided that the antenna is calibrated at each frequency.

DIRECTIVITY AND POWER GAIN

After the angle-of-arrival of a signal has been determined, it is then possible to combine the three receptions with phase adjustments to boost the total received power. Doing that provides a modest amount of antenna directivity in the direction of the desired signal and power gain. The resulting half-power beamwidth is about 70 degrees, and the amount of power gain is about 4 dB.

SUMMARY

Comparing the four-element Adcock antenna, which was used in the original TCAS tests, with a three-element triangle antenna, there is a significant performance advantage for the triangle antenna. The advantage calculated in this document, which applies only to the effects of receiver noise, is an improvement by about 10 dB. Ripples in the transfer function of the Adcock antenna contribute an additional error of about $\pm 8^\circ$ and also degrade the sensitivity to receiver noise in certain directions. In other words, the Adcock antenna is significantly more susceptible to receiver noise.

On the other hand, the Adcock antenna has an advantage in requiring two receivers instead of three. The analysis given here does not include element to element coupling, and it does not include the effects of nearby antennas installed on the same aircraft or the tail or other parts of the aircraft that may degrade angle-of-arrival estimation.

It would also be possible to use the triangle antenna configuration in a ground based installation. If the antenna were mounted on a modest tower or platform well above the ground, and free of nearby obstructions, then the main limitation to azimuth accuracy would be receiver noise, and there would be a very significant advantage in using a triangle antenna instead of an Adcock antenna.

REFERENCES

1. D.A. Spencer et al., "TCAS Experimental Unit (TEU) Hardware Description," MIT Lincoln Laboratory, Lexington, MA, Project Report ATC-133 (6 June 1986), DOT/FAA/PM-85/2.
2. W.H. Harman et al., "Traffic Alert and Collision Avoidance System (TCAS) Surveillance Performance in Helicopters," MIT Lincoln Laboratory, Lexington, MA, Project Report ATC-135 (8 May 1987), DOT/FAA/PM-85/29.

APPENDIX ANALYSIS OF AZIMUTH ACCURACY

The following analysis applies to receptions from a transmitter infinitely far away. Mutual coupling is not included in this analysis, so the different monopole receptions differ from each other only as a consequence of the different path lengths from the transmitter. Phase between two receptions is defined counterclockwise positive, and negative when the second of the two monopoles is farther from the transmitter. For example, phase AB is negative when B is farther from the transmitter.

The use of three phase measurements to make an estimate of azimuth is illustrated in Figure 4. The three phase measurements can be considered to describe a curve in the three-dimensional coordinate system,

$$\begin{aligned}x &= \text{phase AB} \\y &= \text{phase BC} \\z &= \text{phase CA} .\end{aligned}$$

As azimuth varies, the locus of points in the x,y,z coordinate system is seen in Figure 4 to be a closed figure like an ellipse or circle. After rotating the coordinate system, it is found that the locus is a perfect circle in the $x''y''$ plane. There are two rotations, as follows. The first rotation is in the xy plane,

$$\begin{aligned}x' &= x \cos \theta_1 + y \sin \theta_1 \\y' &= -x \sin \theta_1 + y \cos \theta_1 \\z' &= z ,\end{aligned}$$

where $\theta_1 = \pi/4$. The second rotation is in the $x'z'$ plane,

$$\begin{aligned}x'' &= x' \cos \theta_2 + z' \sin \theta_2 \\y'' &= y' \\z'' &= -x' \sin \theta_2 + z' \cos \theta_2 ,\end{aligned}$$

where $\theta_2 = \arctan(-\sqrt{2}) = -0.9553$ radian. Putting the two rotation formulas together, the combined formulas for calculating x'' and y'' from the measurements x,y,z are as follows:

$$\begin{aligned}x'' &= x \cos \theta_1 \cos \theta_2 + y \sin \theta_1 \cos \theta_2 + z \sin \theta_2 \\y'' &= -x \sin \theta_1 + y \cos \theta_1\end{aligned}$$

The locus in the x'',y'' plane is a circle of radius $\pi * \sqrt{(3/2)}$, which equals 3.848 radians. The discrete points in this figure are samples of azimuth over 360° evenly spaced by 10° . The even spacing that appears in the $x''y''$ plane suggests that the polar angle in the $x''y''$ plane can be taken as an estimate of azimuth, which in fact is true. Specifically, in the absence of noise,

$$\text{polar angle} = \arctan(x'',y'') = \text{azimuth} - \pi/6 ,$$

where polar angle is take to be zero in the plus x direction, counterclockwise positive. In other words, azimuth can be estimated by taking the polar angle in the x"y" plane and adding 30 degrees.

Receiver Noise. Each of the receivers illustrated in Figure 2 will contribute noise, which will introduce errors to the azimuth estimate. Analysis of the resulting azimuth accuracy begins by characterizing each receiver as adding a noise of power N to the reception. The receiver noises in different receivers are considered to be independent. The received signal power is denoted S, and therefore the signal-to-noise ratio is S/N. The total reception for one receiver including both signal and noise is denoted r(t),

$$r(t) = s(t) + n(t) ,$$

where s(t) is the signal, n(t) is the noise, and t is time. For simplicity, the signal is considered to be a simple sinewave,

$$s(t) = a \cos(\omega t) ,$$

where a is the amplitude and ω is the radian frequency. Signal power S is related to amplitude a by

$$\text{signal power} = S = \text{average}[s(t)]^2 = a^2/2 .$$

The receiver noise has two components, in-phase and quadrature.

$$n(t) = n_1 \cos(\omega t) + n_2 \sin(\omega t) ,$$

and these two components of noise are independent and have equal variances, σ^2 ,

$$\sigma^2 = \text{variance of } n_1 = \text{variance of } n_2 .$$

Noise power N is related to variance σ^2 by

$$\text{noise power} = N = \text{average}[n(t)]^2 = \sigma^2 .$$

Therefore, the signal-to-noise ratio is related to amplitude a and noise variance σ^2 by

$$S/N = a^2/(2 \sigma^2) .$$

Cases of high signal-to-noise ratio are interesting because they are relatively simple. When S/N is high, only the component of noise in quadrature with the signal causes a phase error. The resulting phase of r(t) will have an error of

$$\text{phase error} = \arctan(n_2/a) \sim n_2/a \text{ (radians)} .$$

It follows that the mean-square phase error equals

$$\text{mean-square phase error} = \text{mean}(\phi^2) = 1/(2 S/N) \text{ (radians}^2\text{)} .$$

This applies to a single receiver. When a phase measurement is made between two receivers, they both contribute errors.

For a triangle antenna, the azimuth estimation process begins by making three relative phase measurements (as shown above).

$$\begin{aligned} x &= \text{phase AB} \\ y &= \text{phase BC} \\ z &= \text{phase CA} \end{aligned}$$

These three values are rotated into x'', y'', z'' values, for which the azimuth estimate is simply the arctangent of (x'', y'') . If the phase errors in receivers A, B, and C are denoted ϕ_A, ϕ_B, ϕ_C , then the x, y, z errors are

$$\begin{aligned} \phi_x &= \phi_A - \phi_B \\ \phi_y &= \phi_B - \phi_C \\ \phi_z &= \phi_C - \phi_A . \end{aligned}$$

Note that the phase error contributed by each receiver affects two of the x, y, z values. Given that the three receiver phase errors are independent, their effects can be combined by summing the mean-square contributions to each of x'' and y'' . It can be shown that the sum is the same for both x'' and y'' .

$$\text{mean square error of } x'' = \text{mean square error of } y'' = 3 * \text{mean}(\phi^2) ,$$

where $\text{mean}(\phi^2)$ is the mean square phase error in one receiver in radians. Furthermore, it can be shown that the errors in x'' and y'' are uncorrelated, and therefore that the total error to the point x'', y'' is an isotropic noise independent of azimuth. Azimuth accuracy is the same in every direction.

If S/N is high, the errors in x'' and y'' are small relative to the radius ($\pi \sqrt{3/2}$ radians). So only the quadrature component of x'', y'' error affects azimuth accuracy. The mean square azimuth error is

$$\text{mean square azimuth error} = 3 * \text{mean}(\phi^2) / (\text{radius})^2 .$$

It was shown above that for one receiver,

$$\text{mean}(\phi^2) = 1/(2 S/N) .$$

Therefore, the mean square azimuth error of the triangle antenna is

$$\text{mean square azimuth error} = [3/(2 S/N)] / (\text{radius})^2 = 1/(\pi^2 S/N) .$$

Adcock Accuracy. This result will be compared with the accuracy of an Adcock antenna in two ways. The first comparison applies to an idealized Adcock antenna, having a perfectly linear phase characteristic, as illustrated in Figure 3. The actual Adcock phase characteristic has some ripples, as shown below, which degrade accuracy somewhat relative to this simpler linear model.

For the linear Adcock model, there are two contributions to phase error, one from each of the two receivers.

$$\text{mean}(\phi^2) = 1/(2 S/N) \text{ each}$$

Therefore, the relative phase error is larger by a factor of 2 in mean square value.

$$\text{mean square relative phase error} = 1/(S/N)$$

The slope of the phase characteristic is a constant 1 (radians of azimuth per radian of phase), so the mean square azimuth error is

$$\text{mean square azimuth error} = 1/(S/N) .$$

These results show that the triangle antenna improves azimuth accuracy by a factor of π^2 in mean square values. Expressed in dB, the improvement is $10 \log(\pi^2) = 9.9$ dB. Expressed in root-mean-square values, the improvement is a factor of π .